THERMAL PROPERTIES OF FRACTAL STRUCTURE MATERIALS

Oldřich Zmeškal, Miroslav Buchníček, Martin Nežádal
Pavla Štefková, Radek Capoušek

Institute of Physical and Applied Chemistry, Faculty of Chemistry, Brno University of Technology, Purkyňova 118, CZ-61200 Brno, Czech Republic
email: zmeskal@fch.vutbr.cz

Abstract

The article is dealing with study of thermal properties of fiber materials. For specific heat, thermal diffusivity and thermal conductivity determination the transient pulse method and step wise were used. The evaluation was carried with the help of mathematical apparatus used for study of properties of fractal structures. The results that were obtained are the same as results obtained by the classical methods.

Key words: fractal structure, specific heat, thermal diffusivity, thermal conductivity, transient pulse method, and stepwise method

1. Introduction

The article is dealing with the description of the new data evaluation method. The method comes out of generalized relations that were designed for the study of physical properties of fractal structures [1, 2]. In the work it is shown that these relations are in a good agreement with the equations used for the description of time responses of temperature for the pulse input of supplied heat [3, 4, 5]. Thermal parameters (specific heat, thermal diffusivity, thermal conductivity) calculated by the both methods are the same.

2. Theory

In papers [1, 2] the density of fractal physical quantity $\rho(r)$, (e.g. specific yield of the heat source $q_0(r)$ in J.m^{-3}.s^{-1} for temperature field) in E - dimensional Euclidean space E_n ($E = n$) was defined

$$q_0(r) = eKr^{D-E}, \tag{1}$$

where r is the radius of elementary quantity, K is a fractal measure and D a fractal dimension. From specific yield of the heat source $q_0(r)$ (1) we can determine density of heat flow rate $\boldsymbol{q}(r)$ (in J.m^{-2}.s^{-1}) and temperature $T(r)$ ($q_0 = \operatorname{div} \boldsymbol{q} = \operatorname{div}(-\lambda \operatorname{grad} T) = -\lambda\, \Delta T$), where λ is constant thermal conductivity (in J.m^{-1}.s^{-1}.K^{-1}). For radial temperature field we can write

$$q_\mathrm{r} = eK \cdot \frac{r^{D-E+1}}{D}, \qquad T_\mathrm{r} = -\frac{eK}{\lambda} \cdot \frac{r^{D-E+2}}{D(D-E+2)}. \tag{2}$$

By integrating of eq. (1) over volume $V^* = r^E$ of E-dimensional space we can calculate the power of heat (in J. s^{-1})

$$Q_0(r) = \int_{V^*} q_0(r)\,\mathrm{d}V^* = eK\frac{E\,r^D}{D} = \frac{E\,r^E}{D} q_0(r), \tag{3}$$

where $\mathrm{d}V^* = \mathrm{d}(r^E)$ is an elementary volume of E-dimensional space.

If we suppose that heat permeates through the surrounding by the constant speed then the r can be observed as the size of the invariant space-time vector $r^2 = c^2 t^2 - r_{\mathrm{T}}^2$, where $r_{\mathrm{T}}^2 = x^2 + y^2 + z^2$ is the magnitude of the position vector and c is the maximum of the heat permeation speed (e.g. heat radiance in the vacuum, speed of light in the vacuum respectively).

$$T_{\mathrm{r}} = -\frac{eK(r_0^2)^{(D-E+2)/2}}{\lambda D(D-E+2)} \cdot \left(1 - \frac{r_{\mathrm{T}}^2}{r_0^2}\right)^{(D-E+2)/2}, \tag{4}$$

where $r_0^2 = c^2 t^2$.

If the heat diffuses by the significantly lesser speed then the critical speed ($r_{\mathrm{T}} << ct$, small distances or long times) the terms in parenthesis can be observed as significant in the expansion of exponential function ($1 - x \approx \mathrm{e}^{-x}$) and we can write then

$$T_{\mathrm{r}} = -\frac{eK(r_0^2)^{(D-E+2)/2}}{\lambda D(D-E+2)} \cdot \exp\left(-\frac{D-E+2}{2} \cdot \frac{r_{\mathrm{T}}^2}{r_0^2}\right). \tag{5}$$

If we substitute for the thermal conductivity $\lambda = c_{\mathrm{p}} \rho a = c_{\mathrm{p}} \rho r_0 / [2c(D-E+2)]$, where c_{p} is specific heat capacity after constant pressure (in J.kg^{-1}. K^{-1}), ρ mass density (in kg.m^{-3}) and a the coefficient of thermal diffusivity of the body (in m^2. s^{-1}), and for the total heat transferred to the body from the heat source $Q = -2eKr_0/(cD) \cdot [2\pi/(D-E+2)]^{E-D}$ we obtain

$$T_{\mathrm{r}} = \frac{Q}{c_{\mathrm{p}} \rho} \left(\frac{2\pi r_0^2}{D-E+2}\right)^{(D-E)/2} \cdot \exp\left(-\frac{D-E+2}{2} \cdot \frac{r_{\mathrm{T}}^2}{r_0^2}\right), \tag{6}$$

resp.

$$T_{\mathrm{r}} = \frac{Q}{c_{\mathrm{p}} \rho (4\pi a t)^{(E-D)/2}} \cdot \exp\left(-\frac{r_{\mathrm{T}}^2}{4at}\right). \tag{7}$$

That is the relation for fractal dimension D = 0, 1, 2 and topological dimension E = 3 published in [3, 4, 5]. The *Figure 2* represents time-temperature dependencies (according *eq.7*) calculated for spherical ($D = 0$), cylindrical ($D = 1$), planar ($D = 2$), and cubic ($D = 3$) geometry of the heat source (see *Figure 1*). The maximum position can be determined by the derivation of equation (7) with the time

$$\frac{\partial \log T_{\mathrm{r}}}{\partial \log t} = \left(\frac{D-E}{2} + \frac{r_T^2}{4at}\right) = 0. \tag{8}$$

It is evident from the *Figure 2* and from the equation (8) that for $D = E$ the function meets maximum for time $t \to \infty$. In the other cases the diffusivity a can be determined from the time when the temperature is maximal

$$a = r_{\mathrm{T}}^2 / [2(E-D)t_{\mathrm{m}}], \tag{9}$$

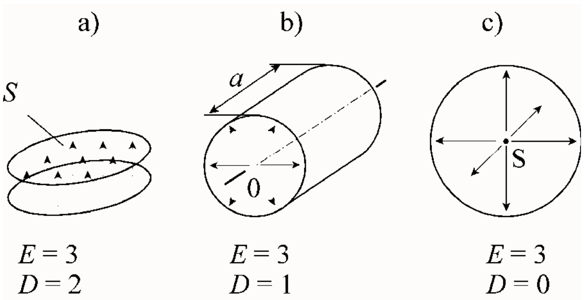

Figure 1 Heat flow geometry for a) plane-parallel, b) cylindrical and c) spherical coordinates Euclidean space

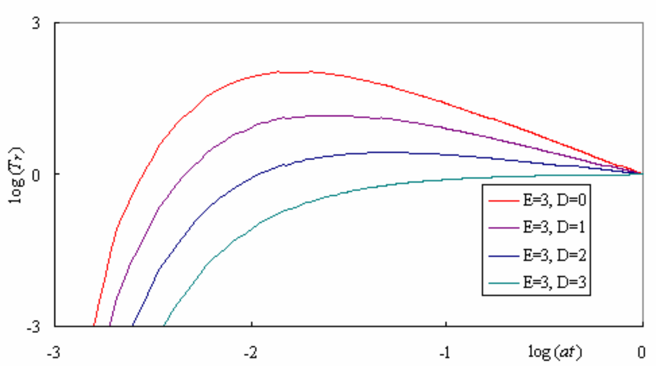

Figure 2 Time dependency of the temperature response for the Dirac thermal pulse (for heat flow geometry from figure 1) calculated by eq. (6)

where $f_a = E - D$ it is a coefficient that characterizes the deformation of the thermal field [5]. If we substitute the value of diffusivity to the equation (7) we obtain the thermal capacity:

$$c_{\mathrm{p}} = \frac{Q}{\rho T_{\mathrm{m}} r_{\mathrm{T}}^{E-D}} \cdot \left(\frac{E-D}{2\pi \exp(1)} \right)^{(E-D)/2} , \qquad (10)$$

thermal conductivity of the studied fractal structure respectively

$$\lambda = c_{\mathrm{p}} \rho a = \frac{Q}{2(E-D) T_{\mathrm{m}} t_{\mathrm{m}} r_{\mathrm{T}}^{E-D-2}} \cdot \left(\frac{E-D}{2\pi \exp(1)} \right)^{(E-D)/2} , \qquad (11)$$

where T_m is the maximum temperature of the response for Dirac thermal pulse.

3. Experimental

For the responses to the pulse heat the Thermophysical Transient Tester 1.02 was used. It was developed at Institute of Physics, Slovak Academy of Science [6]. The block diagram of automated measurement workstation is presented in *Figure. 3*. The measured sample, which was placed in the isothermal chamber, consisted of three parts of cylindrical shape. Between the first and the second part the heat source was placed (20 µm thick nickel folium in kapton, and radius R_2 = 1, 2 or 3 cm, (see on *Figure 4*) and first of differentially connected thermo-couple (NiCr-Ni). Between the second and third part one connection of differentially connected thermocouple (NiCr-Ni) was placed too. The second connections of both thermocouples were placed on heat exchanger where the constant temperature was kept with the help of thermostat. The reference temperature was measured by platinum resistance (Pt100Ω).

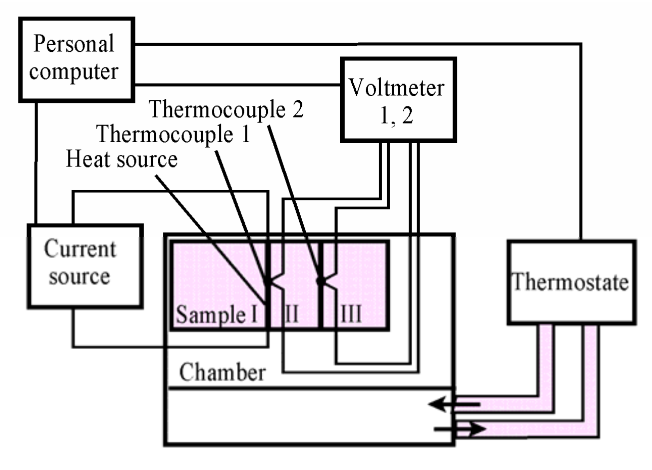

Figure 3 The block scheme of measuring apparatus

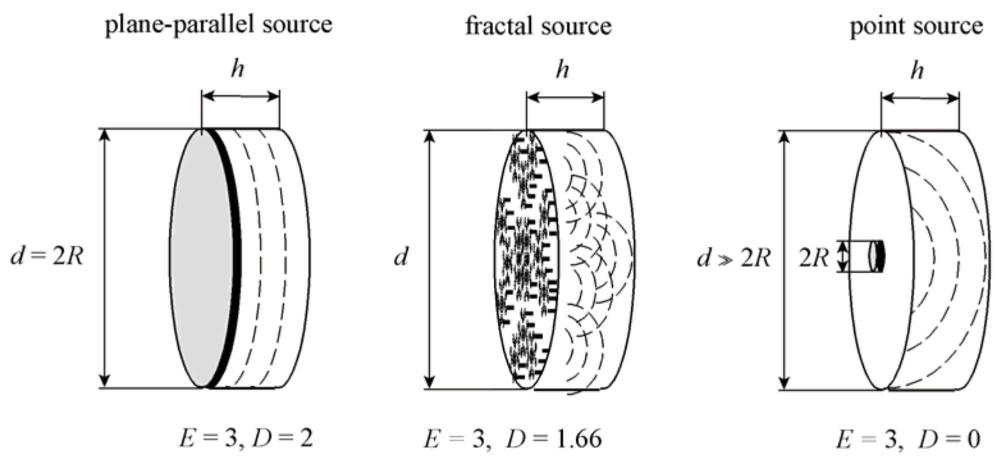

Figure 3 Current flow geometry: a) plane-parallel, B) point (for different ratio of length contact respectively)

Heating-up of sample was provided by rectangular long current pulse from the software directed source Mesit Z-YE-3T/x. The power of supplied heat was computed from the parameters of pulse (from the voltage U and current I)

$$Q = U\,I\,. \tag{12}$$

The changes of temperature between heat exchanger and sample were measured by nanovoltmeter Agilent HP4119A. PC carried the experiment control via GPIB and RS232 bus and software equipment created by authors.

4. Measured samples

Fiber glass wool is a lightweight, flexible, thermal and acoustical insulation material designed to provide the ultimate noise reduction. It is formed from resin-bounded borosilicate glass fibers. It is water and fire resistant, it has low density of combustion gas and low toxicity. It reduces transport of heat and sound. Its density in non-pressed state is 5 – 20 $kg.m^{-3}$, thermal conductivity is 0.03 – 0.04 $W.m^{-2}.K^{-1}$ in 10°C [6].

Samples were round shaped with radius $R_1 = 3$ cm, their thickness was changed by pressing in the range of $h = (30 - 5)$ mm. In this article there are discussed results of measurement by pulse and by stepwise method [7].

5. Results

The *Figure 5* represents the typical responses of temperature for the unitary step of inputted power. The average power of the heat source was 1.17W lasting over the whole time of measurement. (it is approximately 1200s) The upper curve shows the temperature of the heat source; temperature was calculated from the thermal change of its resistance. Its temperature in the steady state relative to the temperature of the heat exchanger (measured by platinum resistance) is 104.7K. The second curve shows course of temperature at the surface of kapton cling film in which is sealed the heat source. It was measured by Thermocouple 1. Between the resistance and the surface of kapton cling film there was 0.65K difference. The third dependency expresses the course of temperature on the opposite side of the sample. It was measure by the Thermocouple 2. The difference of the steady state temperatures relative to heat source is 75.49K in this case. By the derivation of the temperature response (*Figure 6*) we obtain

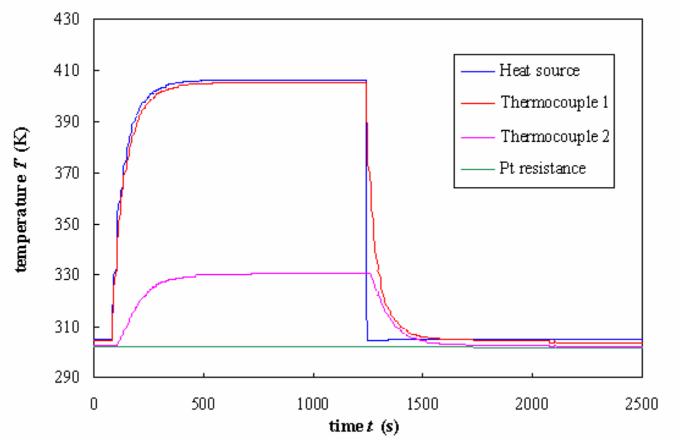

Figure 5 Thermal response of fiber sample for current pulse of 1.17W and duration 1200 s

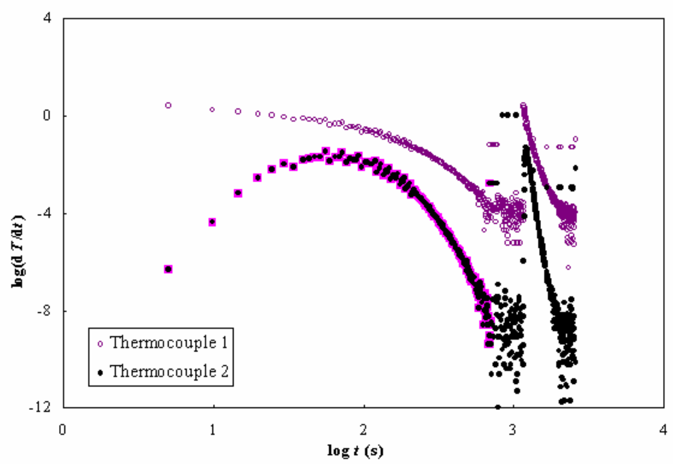

Figure 6 Derivation of thermal response of fiber sample for current pulse of 1.17W and duration 1200 s

temperature response for the Dirac thermal pulse. In the left part of the figure there are showed courses of temperature on the both sides of sample after the switching the heat source on. In the right part of the figure there are responses for switching the heat source off. From these relations the diffusivity can be calculated (according to eq.8]. Its value is presented in the *Table 1* together with the value obtained by pulse method [7]

Table I The comparison of the results of pulse and step-wise method

method	h (mm)	Δt_m (s)	ΔT_m (K)	f_a [5] $(E-D)$	f_c [5]	λ (W.m^{-1}.K^{-1})	c (J.kg^{-1}.K^{-1})	a (m^2.s^{-1})
pulse	8	58.14	7.49	1.879	0.610	0.0427	3460	$2.93 \cdot 10^{-7}$
step	8	55.55	5.33	1.879	0.610	0.0414	3419	$3.07 \cdot 10^{-7}$

6. Conclusion

In this article there are presented results of measurements of thermal parameters (thermal conductivity, specific heat and thermal diffusivity) of glass wool fibers. In the theoretical part there are presented equations for computing parameters of thermal systems in fractal structures. Obtained equations are compared with equations used for evaluation with the help of pulse and step-wise method [5]. By these equations the experimental values of these parameters were calculated. The congruence in the theory and the experimental shows, that suggested mathematical apparatus is suitable for study of thermal properties of structures with the fractal structure.

References

[1] Zmeškal O., Nežádal M., Buchníček M.: Fractal–Cantorian Geometry, Hausdorff Dimension and the Fundamental Laws of Physics. Chaos, Solitons & Fractals 2003; 17: 113–119

[2] Zmeškal O., Nežádal M., Buchníček M.: Field and Potential of fractal–Cantorian structures and El Naschie's infinite theory. Chaos, Solitons & Fractals 2004; 19: 1013–1022

[3] Carslaw H.S., Jaeger J. C.: Conduction of Heat in Solids. Clarendon Press London 1959, 496 pp.

[4] Krempaský J.: Measurement of Thermophysical Quantities. VEDA Bratislava 1969, 287 pp.

[5] Kubičár L.: Pulse Method of Measuring Basic Thermophysical Parameters, VEDA Bratislava and Elsevier Nederland 1990, 344 pp.

[6] Thermophysical Transient Tester – Model RT 1.02, Institute of Physic, Slovak Academy of Sciences

[7] Zmeškal O., Nežádal M., Lapčík L.: Thermal Conductivity of Borosilicate Glass Wool, Thermophysics 2002, Kočovce 2002.