SENSITIVITY COEFFICIENTS ANALYSIS IN EDPS METHOD

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Abstract

This work reports on the method for measuring thermophysical properties (thermal conductivity and diffusivity) of materials. The theory of the dynamic plane source method and experimental apparatus is described. A new algorithm for sensitivity coefficient analysis is presented and the results are compared with those of difference analysis of experiment modelling.

Key words: thermophysical properties, dynamic plane source method, difference analysis, sensitivity coefficient

1 Introduction

Development of new materials and advancement of material engineering has influenced the development of measuring methods of their physical properties in last decades. Thermophysical properties belong to the most important material properties. Progress of the electronics and computer technology has caused transition from stationary to unstationary methods. Transient methods [1] are based on generation of a dynamic temperature field inside the specimen. The measuring process can be described as follows. The temperature of the specimen is stabilised and uniform. Then the dynamic heat flow in the form of pulse or step-wise function is generated inside the specimen. From the temperature response to this small disturbance the thermophysical parameters of the specimen can be calculated.

2 Experimental

The extended dynamic plane source (EDPS) method is arranged for a one-dimensional heat flow into a finite sample. The configuration of the experiment is obvious from Figure 1. The plane source (PS) disc, which simultaneously serves as the heat source and thermometer, is made of a nickel film covered from both sides with kapton layer. The heat in the form of a step-wise function is produced by the passage of the electrical current through a PS disc. Two identical samples in the cylindrical shape cause symmetrical division of the heat flow into a very good heat conducting material (heat sink), which provides isothermal border conditions of an experiment. This method appears to be useful for simultaneous determination of thermal diffusivity a and thermal conductivity λ of low thermally conducting materials.

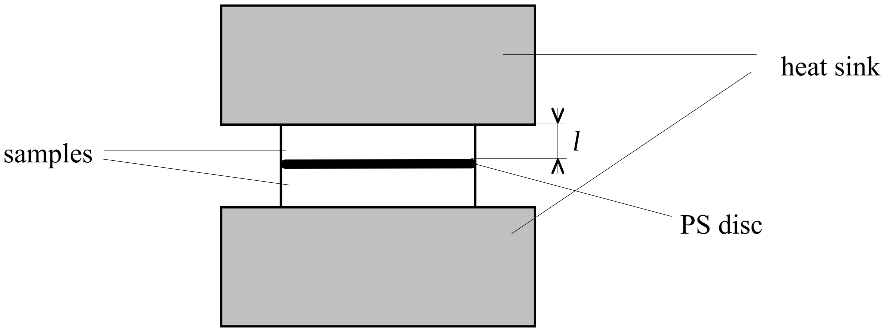

Figure 1. The setup of the experiment.

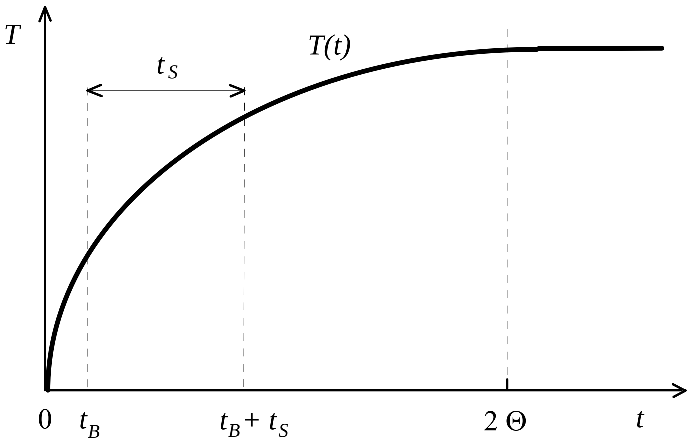

Figure 2. Temperature function - temperature increase as a function of time.

Figure 2 shows the theoretical temperature function which is a solution of the partial differential equation with boundary and initial conditions corresponding to the experimental arrangement. The temperature function is given by [2]

$$T(t) = \frac{ql}{\lambda\sqrt{\pi}} F(\Theta, t) \ , \tag{1}$$

where

$$F(\Theta, t) = \sqrt{\frac{t}{\Theta}} \left(1 + 2\sqrt{\pi} \sum_{n=1}^{\infty} \beta^n \operatorname{ierfc}\left(n\sqrt{\frac{\Theta}{t}} \right) \right). \tag{2}$$

q is the heat current density, λ is the thermal conductivity and Θ is the characteristic time of the sample given by

$$\Theta = l^2 / a \, , \tag{3}$$

where l is the thickness and a the thermal diffusivity of the specimen. Parameter β describes the heat sink imperfection and ierfc is the error function integral [3].

The principle of the method is based on fitting the theoretical temperature function over the experimental points. The fitting procedure is based on a linear regression [2,4]. The plot of experimental points T_i versus $F(\Theta, t_i)$, calculated using equation (2), should be a straight line if Θ has its proper value. Equation (1) predicts a zero intercept but real measurements showed non zero value τ referred to the additional increase in the temperature of the PS disc due to its imperfections. The proper value of Θ can be found by using an iteration procedure so that we will change the characteristic time Θ until the correlation coefficient calculated from T_i and $F(\Theta, t_i)$ reaches its maximum. The slope of this straight line gives λ while the iterated Θ gives *a*.

3 Sensitivity coefficients analysis

The sensitivity coefficient is a measure of the change in temperature due to the variation of the estimated parameters. The sensitivity coefficient β_p is defined by [4]

$$\beta_p = p \frac{\partial T(t)}{\partial p}, \tag{4}$$

where *p* is the parameter to be analysed and $T(t)$ is the temperature function. The fitting procedure does not work properly when sensitivity coefficients are low or linearly dependent on each other. Therefore an analysis of the sensitivity coefficients determines the time window in which the evaluation technique can be applied to the temperature response.

In this section we will concentrate to the investigation of the linear dependence of the sensitivity coefficients. As mentioned in the previous section, there are three parameters which values should be estimated. They are two thermophysical parameters of the materialλ, *a* and the baseline of the temperature function τ. Hence, the temperature function (1) can be expressed

$$T(t,a,\lambda,\tau) = \frac{q}{\lambda}\sqrt{\frac{ta}{\pi}}\left(1 + 2\sqrt{\pi}\sum_{n=1}^{\infty}\beta^n \operatorname{ierfc}\left(\frac{nl}{\sqrt{at}}\right)\right) + \tau \tag{5}$$

and the sensitivity coefficients β_a, β_λ and β_τ can be calculated using the formula (4). Figure 3 shows the temperature function and the sensitivity coefficients β_λ and β_a as a function of time. The third coefficient acquires constant value $\beta_\tau = \tau$. Since the sensitivity coefficients are functions of one variable *t*, the linear dependence can be investigated using Wronsky's determinant [5] given by the form

$$W(t) = \begin{vmatrix} \beta_a & \beta_\lambda & \beta_\tau \\ \beta'_a & \beta'_\lambda & \beta'_\tau \\ \beta''_a & \beta''_\lambda & \beta''_\tau \end{vmatrix} . \tag{6}$$

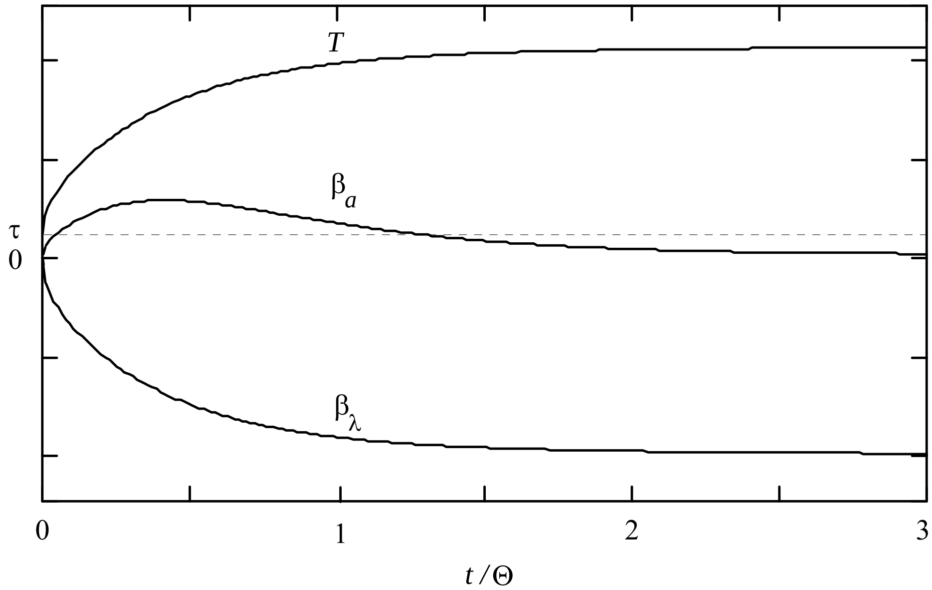

Figure 3. Temperature function $T(t)$ and sensitivity coefficients β_λ and β_a.

The sensitivity coefficients are linearly dependent when the determinant (6) is equal to zero. If the functions are represented by equi-spaced time series [6] the derivations can be estimated by the relation

$$f_i' = \frac{f_{i+1} - f_i}{\Delta t}, \tag{7}$$

where Δt is the time interval between samples. Then the determinant W obtains very simple form

$$W_i = c \cdot \begin{vmatrix} \beta_{a i} & \beta_{\lambda_i} & 1 \\ \beta_{a i+1} & \beta_{\lambda_{i+1}} & 1 \\ \beta_{a i+2} & \beta_{\lambda_{i+2}} & 1 \end{vmatrix}, \tag{8}$$

where c is a constant. Function (8) in the form of a time series determines the time interval, where the sensitivity coefficients are not linearly dependent. As seen in Figure 4 function W acquires non zero values in the interval $(0.07\Theta, \Theta)$.

4 Results and discussion

In order to verify the theory described in the preceding section we decided to construct a mathematical model of the experiment. In the first stage the points were computed using equations (1-3). Simulating the measurement of PMMA, the following parameter values were used: l = 2.86 mm, q = 1053 W/m^2, λ = 0.19 W/mK, a = 0.12 10^{-6} m^2/s and β = -0.954. The measuring error was added by rounding the temperature coordinate of the points to 4 valid numbers. Then the points were processed by difference analysis [7], which is based on fitting the theoretical temperature function (1) to the points in the time interval $(t_B, t_B + t_S)$, where t_B and t_S designate the beginning and the size of the interval, respectively (Fig.2). If t_B is successively changed while t_S is keeping constant a series of parameter values are obtained. Figure 4 shows the plot of the relative differences, which are defined by the formula

$$R_x = \left| \frac{x - x_0}{x} \right| , \tag{9}$$

where x_0 is the value put originally into the model and x is the value calculated using fitting procedure. If the time interval $(t_B, t_B + t_S)$ is not suitable for parameters a and λ estimation, the results of fitting are erroneous and relative differences are far from zero value.

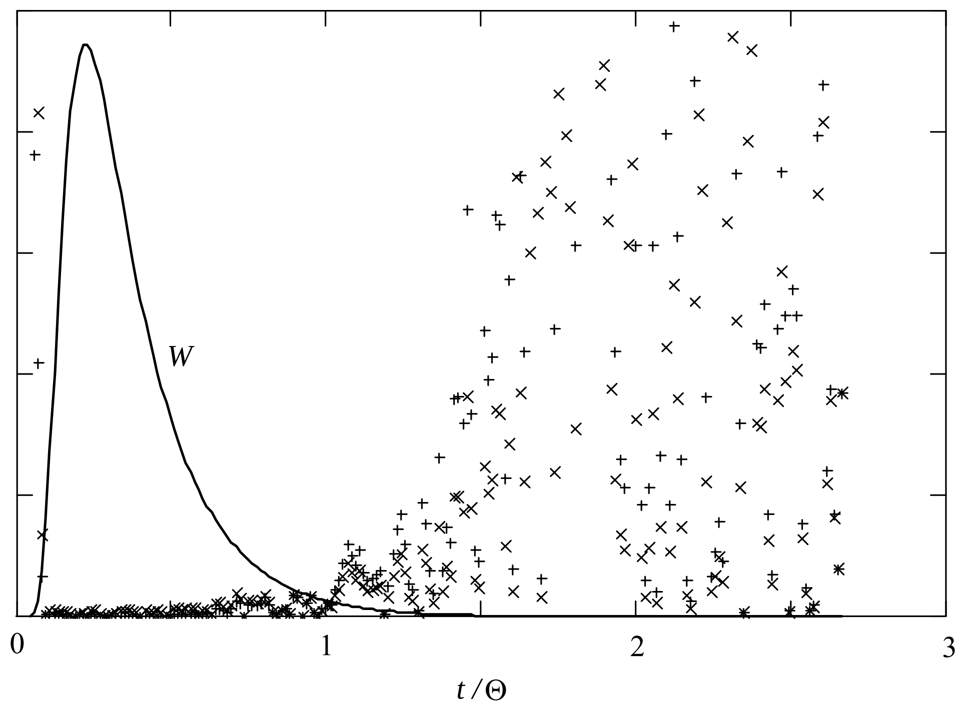

Figure 4. The values of determinant *W* and relative differences of the parameters *a* (x) and λ (+).

Figure 4 illustrates the excellent consonance between sensitivity coefficient analysis results, represented by the function *W*, and difference analysis, represented by relative differences of both parameters *a* and λ. In the interval $(0.07\Theta, \Theta)$ determinant *W* acquires non zero values, so that the sensitivity coefficients are not linearly dependent, fitting procedure works properly and computed values are nearly the same as values put originally into the model. The lower the values of determinant *W*, the higher the scattering of computed values of thermophysical parmeters *a* and λ.

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