Moisture Diffusivity Estimation by Temperature Response

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Abstract

This paper deals with the solution for the inverse problem of parameter estimation involving heat and mass transfer in capillary porous media.

The present parameter estimation problem is solved with the Levemberg-Marquardt algorithm of minimization of the leas-squares norm, by using only temperature experimental data. The objective is to identify moisture diffusivity coefficient as function of moisture. The temperature responses are obtained with a numerical solution of the non-linear one-dimensional Luikov`s equations.

Keywords: heat and moisture transfer, inverse problem, moisture diffusivity

Introduction

The phenomena of coupled heat and mass transfer in capillary porous media has been drawing the attention of research groups for a long time, because of its importance in several practical applications. For the mathematical modeling of such phenomena, Luikov [1] has proposed a model based on system of coupled partial differential equations, which takes into account the effects of the temperature gradient on the moisture migration.

The system of equations incorporates coefficients that must be determined experimentally. The main problem is the determination of the moisture diffusivity content measurements. Local moisture content measurements are practically unfeasible especially for small drying objects. The objective of this paper is to determine the moisture diffusivity coefficient by application of inverse analysis approaches. The main idea of the applied method is to take advantage of the relation between the heat and moisture transport process within the drying body and from its surface to the surrounding media. Then, the estimation of the moisture diffusivity coefficient of the drying body could by performed on basis of accurate and easy-to-perform thermocouple temperature measurements.

Mathematical model of the problem

The physical problem involves a one-dimensional convective drying experiment of capillary porous sample, initially at uniform temperature and uniform moisture content. One of the boundaries, which one is impervious to moisture, is in direct contact with heater. The other boundary is in contact with the dry surrounding air, thus resulting in a convection boundary condition for both temperature and the moisture content.

The governing partial differential equations, for the modelling of such physical problem, are derived from conservation of mass and energy flow in a 1-D element volume of porous material by Luikov [1]. These are written as

$$\rho c_m(T,u)\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\lambda(T,u)\frac{\partial T}{\partial x}\right)-l(T)\frac{\partial}{\partial x}\left(a_u(T,u)\frac{\partial u}{\partial x}\right) \tag{1}$$

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(D_u(T,u)\frac{\partial u}{\partial x} + D_T(T,u)\frac{\partial T}{\partial x}\right) \qquad (2)$$

where T is temperature , u is volume basic moisture content, ρ is density solid matrix, c_m is specific heat of sample, λ is thermal conductivity of sample, ρ_m is the water density, l is latent heat of vaporization, D_u is moisture transport coefficient associated to moisture content gradient, D_T and a_u are transport cross coefficients.
Boundary conditions are expressed as follows

$$\left(\lambda(T,u)\frac{\partial T}{\partial x}\right)_{x=0} = -q(t) \qquad \text{for } t > 0 \qquad (3)$$

$$\left(D_u(T,u)\frac{\partial u}{\partial x} + D_T(T,u)\frac{\partial T}{\partial x}\right)_{x=0} = 0 \qquad \text{for } t > 0 \qquad (4)$$

$$\left(\lambda(T,u)\frac{\partial T}{\partial x}\right)_{x=L} + \left(l(T)a_u(T,u)\frac{\partial u}{\partial x}\right)_{x=L} = h(T_\infty - T_{x=L}) + l(T)h_m(u_\infty - u_{x=L}) \qquad \text{for } t > 0 \qquad (5)$$

$$\left(D_u(T,u)\frac{\partial u}{\partial x} + D_T(T,u)\frac{\partial T}{\partial x}\right)_{x=L} = \frac{h_m}{\rho_m}(u_\infty - u_{x=L}) \qquad \text{for } t > 0 \qquad (6)$$

where $h(T_\infty - T_{x=L})$ represents the heat exchanged with the ambient air and $h_m(u_\infty - u_{x=L})$ is the phase change energy term, h is surface conductance, h_m is mass convection coefficient.
Initial conditions are

$$T(x,0) = T_0 \qquad \text{at} \qquad x \in <0, L> \qquad (7)$$
$$u(x,0) = u_0 \qquad \text{at} \qquad x \in <0, L> \qquad (8)$$

Considering the temperature range of interest in building applications, temperature dependence of lime mortal cross transport coefficients is, here, neglected when compared to their moisture content dependence. The objective is to determine coefficient $D_u(u)$.

Direct problem

The mathematical model is discretized by using the control-volume method and the interpolation is realized by the central-difference scheme.

$$\left(\rho_0 c_{k,i}\frac{\Delta x_i}{\Delta t_k} + \frac{\lambda_{k,i-1}}{\Delta x_{i-1}} + \frac{\lambda_{k,i+1}}{\Delta x_{i+1}}\right) T_{k,i} = \frac{\lambda_{k,i}}{\Delta x_{i-1}} T_{k.i-1} + \frac{\lambda_{k,i+1}}{\Delta x_{i+1}} T_{k,i+1} + \rho_o c_{k-,i}\frac{\Delta x_i}{\Delta t_{k-1}} T_{k-1,i} + \mathrm{l}_{\mathrm{k-1,i-1}}\rho_l \left(\frac{D_{u,k-1,i-1}\left(u_{k-1,i-1} - u_{k-1,i}\right)}{\Delta x_{i-1}} - \frac{D_{u,k-1,i+1}\left(u_{k-1,i} - u_{k-1,i+1}\right)}{\Delta x_{i+1}}\right) \qquad (9)$$

$$\left(\frac{\Delta x_i}{\Delta t_k}+\frac{D_{u,k,i-1}}{\Delta x_{i-1}}+\frac{D_{u,i+1}}{\Delta x_{i+1}}\right)u_{k,i}=\frac{D_{u,k.i-1}}{\Delta x_{i-1}}u_{k,i-1}+\frac{D_{u,k,i+1}}{\Delta x_{i+1}}+\frac{\Delta x}{\Delta t_{k-1}}u_{k-1,i}+ \\ \left(\frac{D_{T,k-1,i-1}}{\Delta x_{i-1}}\left(T_{k-1,i-1}-T_{k-1,i}\right)-\frac{D_{T,k-1,i+1}}{\Delta x_{i+1}}\left(T_{k-1,i}-T_{k-1,i+1}\right)\right) \tag{10}$$

where $k=0,1,2,...K$ refers to time step ($k=0$ refers to the initial condition) and $i=1,2,3,...,I+1$ refers to spatial grid points ($i=1$ and $i=I+1$ refers to the boundaries). In the same way, boundary conditions are put in discrete form to obtain the following expressions for the half volume node at the boundaries ($x=0$ and $x=L$)

$$\left(\rho_0 c_{k,i-1}\frac{\Delta x}{2\Delta t}+\frac{\lambda_{k,i-1}}{\Delta x}+\frac{L\rho_l D_{Tvk,i-1}}{\Delta x}+h\right)T_{k,0}=\left(\frac{\lambda_{k,i-1}}{\Delta x}+\frac{L\rho_l D_{Tv,i-1}}{\Delta x}\right)T_{k,1}+ \\ \rho_o c\frac{\Delta x}{2\Delta t}T_{k-1,0}+L\rho_l D_{uv,k,i-1}\left(\frac{u_{k-1,1}-u_{k-1,0}}{\Delta x}\right)+hT_\infty+Lh_m\left(\rho_{v,\infty}-\rho_{v,0}\right) \tag{11}$$

$$\left(\frac{\Delta x}{2\Delta t}+\frac{D_{u,k,i-1}}{\Delta x}\right)u_{k,0}=\frac{D_{u,k,i-1}}{\Delta x}u_{k,1}+\frac{\Delta x}{2\Delta t}u_{k-1,0}+D_{T,k,i-1}\left(\frac{T_{k-1,1}-T_{k-1.0}}{\Delta x}\right) \\ +\frac{h}{\rho_l}\left(\rho_{v,\infty}-\rho_{v,0}\right) \tag{12}$$

Inverse procedure

For inverse problem of interested here, moisture diffusivity coefficient $D_u(u)$ is regarded as the unknown quantity, but everything else in direct problem equations is known. For determining of diffusivity coefficient, we assuming that the function $D_u(u)$ is taken as $D(u)=D_0.\exp[b(u-u_0)]$ [2] and we consider available the transient temperature measurements Y_{im} taken at two locations $(m=1,2)$ within the sample. The subscript i refers to the time at which the measurements are taken $(i=1,2,...,I)$. The estimation methodology used is based on minimization of the ordinary least square norm

$$\mathbf{S}(\mathbf{P})=\left[\mathbf{Y}-\mathbf{T}(\mathbf{P})\right]^{\mathrm{T}}\left[\mathbf{Y}-\mathbf{T}(\mathbf{P})\right] \tag{13}$$

here, $\mathbf{P}[P_1,P_2,...,P_N]$ denotes vector of unknown parameters. $[\mathbf{Y}-\mathbf{T}(\mathbf{P})]^T$ is given by $[\mathbf{Y}-\mathbf{T}(\mathbf{P})]^T\equiv[(\vec{Y}_1-\vec{T}_1),(\vec{Y}_2-\vec{T}_2),...,(\vec{Y}_I-\vec{T}_I)]$
where $(\vec{Y}_i-\vec{T}_i)$ is row vector containing the differences between the measured and estimated temperatures in time t_i.
A version of Levenberg-Marquardt method [3] was applied for the solution of the presented parameter estimation problem. The solution for vector $\mathbf{P}$ is achieved using the following iterative procedure

$$\mathbf{P}^{r+1}=\mathbf{P}^r+[(\mathbf{J}^r)^T\mathbf{J}^T+\mu^r\mathbf{I}]^{-1}(\mathbf{J}^r)^T[\mathbf{Y}-\mathbf{T}(\mathbf{P}^r)] \tag{14}$$

where $\mathbf{J}$ is sensitivity coefficient

$$\mathbf{J}(\mathbf{P}) = \begin{bmatrix} \frac{\partial T_1^T}{\partial P_1} & \cdots & \frac{\partial T_1^T}{\partial P_N} \\ . & & . \\ \frac{\partial T_I^T}{\partial P_1} & \cdots & \frac{\partial T_I^T}{\partial P_N} \end{bmatrix} = \left[\frac{\partial T^T(\mathbf{P})}{\partial \mathbf{P}} \right]$$

The term $\mu\mathbf{I}$ damps instabilities due to ill-conditioned character of the problem. So the matrix $\mathbf{J}^T\mathbf{J}$ is not required to be non-singular at the beginning of iterations and the procedure tends towards a slow-convergent steepest decent method. The present iterative procedure stops if the norm of gradient of $\mathbf{S}(\mathbf{P})$ is sufficiently small, or if the changes in the vector of parameters $\mathbf{P}^{\mathbf{r+1}} - \mathbf{P}^{\mathbf{r}}$ are very small [3]. The subroutine OBCLSJ of the IMSL [4] was used in the present work.

Results

By following the same approach of reference [5], we consider the applied heat flux $q(t)$ to be in the form of a step function in time, that is, $q(t) = q_0$ for $0 < t < t_h$ and $q(t) = 0$ if $t > t_h$ (15)

Because the heat flux given by (15) is a piecewise constant function, the solution technique for direct problem needs to be sequentially for the heating period $0 < t < t_h$ and then for the post-heating period.

Let us consider in this paper the test-case involving the following values of material properties for lime mortar: $\rho_0 = 2050.0\,(\mathrm{kg/m^3})$, $c_m = 950 + 0.041\mathrm{T} + 38\mathrm{u}$ (J/kgK), $\lambda = 0.58 + 0.003\mathrm{T} + 0.4\mathrm{u}$ (W/Km), $h = 0.32\,(\mathrm{W/km^2})$, $h_m = 4.7 \cdot 10^3\,(\mathrm{kg/m^2}s)$, transport cross coeffcients $D_T = (3.02\,.10^{-8} + 1.72\,.10^{-7}u)\,\mathrm{m}^2.s^{-1}.K^{-1}$. We present in Table 1 results obtained for the estimated parameters for standard deviation $\sigma = 0.01\,\mathrm{T_{max}}$. Normalised standard deviations are computed by dividing the original standard deviations by the maximum measured temperature.

Table1.Estimated parameters

Experimental conditions	Parameter	Start guess	Estimated parameter	Normalised stand. deviation
$q_0 = 40\ \mathrm{W \cdot m^{-2}}$	D_0	1	1.0312×10^3	0.0013
$\mathrm{t_h} = 40\,\mathrm{min}$	b	7	3.1312×10^{-1}	0.0037
$\mathrm{t_f} = 60\,\mathrm{min}$	u_0	4	3.1123×10^{-1}	0.0085

Conclusion

Moisture diffusivity coefficient in drying process of porous material is important parameter. Precise determination of this experimentally is exhausting work. Presently proposed inverse method has shown a possibility of determining moisture diffusivity coefficient of porous material as function moisture content, only by the temperature measurements. This result is agreed to the results in [2], which were determined experimentally.

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