APPLICATION OF EFFECTIVE MEDIA THEORY IN THE DETERMINATION OF THERMAL CONDUCTIVITY OF WET LIME-POZZOLANA RENDERS

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Abstract:
Moisture dependent thermal conductivity of lime-pozzolana based render is studied in the paper. First, the measurements of thermal conductivity are performed in dependence on moisture content from the dry state to the fully water saturated state using an impulse technique. Then, the obtained data are analyzed using several different homogenization techniques, among them, Lichtenecker, Dobson and Polder and van Santen formulas for various shapes of the pores and inclusions were used. On the basis of this analysis, the most suitable mixing formula giving the best agreement with the experimental data for all studied materials is identified and recommendations for its practical application and possible extensions and limitations to the other types of materials are formulated.

Keywords:
Thermal conductivity, homogenization techniques, lime-pozzolana renders

INTRODUCTION

Thermal conductivity as the main parameter describing the heat transport of building materials appears to be of particular importance for their practical applications. For their use in building structures there is necessary to take into account that their thermal performance is strictly dependent on total pore volume, distribution and cross connections of pores. In materials research, thermal conductivity of dry materials is mainly studied. However, absolutely dry materials never occur in the conditions of building sites. Also the materials already inbuilt in the structures and exposed to the climatic loading exhibit the dependence of their properties on moisture changes. If the material is wet, heat transferred by moisture in the capillaries adds to the density of heat flow rate. The thermal conductivity of water is 0.60 W/mK [1], which is more than 20 times higher than of the air. Therefore, if water is present in the pore space, its effect competes with the effect of air, and the thermal conductivity of a composite material can be considered as a result of this competition, together with the effect of the solid matrix [2]. On this account, there is necessary to have information on the dependence of thermal conductivity on moisture content rising. Experimental measurement of thermal conductivity of several samples having different moisture content is quite time consuming and in consequence expensive, new approaches for the assessment of moisture dependent thermal conductivity have to be explored and tested in materials research.

Homogenization theories working with the concept of an effective medium have proven very useful in a variety of applications in mechanics and in the theory of electricity and magnetism where they already belong to well established treatments (see, e.g., [3]). Within the last couple of years, some references appeared on using the effective media theories for estimation of thermal conductivity of refractory materials, foams, and polymer-based composites. In spite of very promising results, the use of homogenization theory for the assessment of thermal conductivity of lime-based composite materials is still exceptional until now.

In this paper we refer about application of homogenization theory for the assessment of the moisture dependent thermal conductivity of lime-pozzolana composite materials. The measured values of thermal conductivity are analyzed using several types of homogenization formulas originally derived for application in electromagnetic field theory taking into account the limiting bounds of the function of effective thermal conductivity.

EXPERIMENTAL METHODS AND STUDIED MATERIALS

Thermal conductivity as a main parameter describing the heat transport in materials can be experimentally accessed either by steady state methods or by transient methods. For the measurements presented in this work, the commercially produced device ISOMET 2104 (Applied Precision, Ltd.) was used as a typical representative of transient impulse methods. ISOMET 2104 is a multifunctional instrument for measuring thermal conductivity, thermal diffusivity, and volumetric heat capacity. It is equipped with various types of optional probes whereas for our experiments we have chosen surface probe. The measurements in this paper were done in laboratory conditions at average temperature 23 ± 1°C. The material samples were first dried and after that exposed to liquid water for specific time intervals. In this way, the different moisture content of the studied samples was reached. The sample size for thermal conductivity measurement was 70 x 70 x 70 mm.

We have tested newly developed lime-pozzolana based plaster that should find use in restoration and reconstruction of historical buildings (denoted VOM) and reference pure lime plaster (VO). The composition of the studied materials is presented in Table 1.

Table 1. Composition of the studied lime based renders

Type of mixture	Amount in kg			Water
	Lime hydrate	Silica sand 0/2 mm, Bratčice	Metakaoline	
VO	4.80	14.40	-	4.80
VOM	4.80	14.40	0,8	4.80

Plaster mixtures were prepared using laboratory mixing machine with forced rotation for 3 minutes and then compacted using vibrating machine. Each mixture was cast into a cube moulds. After two days were the samples unmoulded and then cured for 28 days in high relative humidity environment.

Both the thermal properties and the moisture content of porous building materials depend mainly on the pore structure of the particular material. Therefore, basic material parameters of the studied materials were measured as well. Nominally, bulk density ρ_b [kg m^{-3}], matrix density ρ_{mat} [kg m^{-3}], and total open porosity ψ [%] were determined. Bulk density was determined on gravimetric principle, matrix density using Pycnomatic ATC. Total open porosity was then calculated from these two quantities. The measured values of basic material parameters are given in Table 2. Since the total porosity of the tested materials is almost the same, the similar thermal performance of the investigated materials can be expected.

Table 2. Basic material parameters of the studied materials

Material	ρ_b [kg m^{-3}]	ρ_{mat} [kg m^{-3}]	ψ [%]
VO	1650	2605	36.7
VOM	1695	2620	35.4

HOMOGENIZATION THEORY

In terms of effective media theory, a porous material can be considered basically as a mixture of three phases, namely solid, liquid and gaseous phase. In the lime based renders studied in this work, the solid phase is represented by the products of joint hydration of lime hydrate, metakaoline and silica sand, the liquid phase by water and the gaseous phase by air. On that account, the homogenization procedure presented in this work was performed in two steps.

The first task was the determination of thermal conductivity of the lime and pozzolana-lime based matrix. This was done on the basis of the known thermal conductivities and amounts of its constituents. In this work, the thermal conductivity of solid matrix was calculated using the Rayleigh [4] mixing rule

$$\frac{\lambda_M - 1}{\lambda_M + 2} = f_c\left(\frac{\lambda_c - 1}{\lambda_c + 2}\right) + f_m\left(\frac{\lambda_m - 1}{\lambda_m + 2}\right) + f_{cf}\left(\frac{\lambda_{cf} - 1}{\lambda_{cf} + 2}\right) + f_w\left(\frac{\lambda_w - 1}{\lambda_w + 2}\right), \tag{1}$$

where λ_M is the thermal conductivity of the whole solid matrix, λ_s thermal conductivity of silica aggregates (3.5 W/mK), λ_c thermal conductivity of hydrated lime – calcium carbonate (5.5 W/mK), f_s volumetric fraction of silica aggregates, f_c volumetric fraction of calcium carbonate. The values of thermal conductivities of particular components of lime-based composites were taken from CRC Handbook of Chemistry and Physics [1]. Because the data on thermal conductivity of metakaoline are not presented in literature, the effect of metaoline addition was involved into the homogenization procedure within the evaluation of the effective thermal conductivity of the whole dry lime based materials on the basis of the comparison of measured and calculated data. Using this backward analysis, the effect of thermal conductivity of metakaoline on the thermal conductivity of pozzolana-lime based composite was identified and considered in homogenization modeling.

The second step within the homogenization procedure represents evaluation of effective thermal conductivity of the whole material, where the mixing is performed for solid matrix, air, and water. As stated in literature, the function of effective thermal conductivity cannot exceed the bounds given by the thermal conductivities and volumetric fractions of its constituents. Several different bounds was already formulated and tested, especially in the theory of electromagnetic field. In this paper, we used for the verification and validation of obtained results Wiener's [5] and Hashin-Shtrikman's [6], [7] bounds.

The upper Wiener's bound is reached in a system consisting of plane-parallel layers of material constituents disposed along the heat flux vector. The lower Wiener's bound is reached in a similar system but with the layers perpendicular to the heat flux. The Wiener's bounds are expressed by the following equations

$$\lambda_{eff} = f_1\lambda_1 + f_2\lambda_2 + f_3\lambda_3, \tag{2}$$

$$\lambda_{eff} = \frac{1}{\frac{f_1}{\lambda_1} + \frac{f_2}{\lambda_2} + \frac{f_3}{\lambda_3}}, \qquad (3)$$

where Eq. (2) represents the upper limit and Eq. (3) the lower limit of the effective thermal conductivity (f_j is the volumetric fraction of the particular phase, λ_j its thermal conductivity, and λ_{eff} effective thermal conductivity of the whole porous body).

Hashin-Shtrikman's bounds were originally derived for two-phase systems only. However, we have performed their extension to three- and four- phase systems. The lower limit of the effective thermal conductivity function can be expressed as

$$\lambda_{lower} = \lambda_1 + \frac{3\lambda_1}{\frac{1}{\sum_{i=2}^{n} f_i \frac{\lambda_i - \lambda_1}{2\lambda_1 + \lambda_i}} - 1}, \qquad (4)$$

and the upper limit as

$$\lambda_{upper} = \lambda_n + \frac{3\lambda_n}{\frac{1}{\sum_{i=1}^{n-1} f_i \frac{\lambda_i - \lambda_n}{2\lambda_n + \lambda_i}} - 1}. \qquad (5)$$

In Eqs. (4)-(5), $f_1 - f_n$ are the volumetric fractions of the particular phase ($f_1 + f_2 + ... + f_n = 1$), and $\lambda_1 - \lambda_n$ are their thermal conductivities, whereas $\lambda_1 < \lambda_2 ... < \lambda_n$.

The second step within the homogenization procedure represents evaluation of effective thermal conductivity of the whole material, where the mixing is performed for solid matrix, air, and water.

For the evaluation of effective thermal conductivity of the whole material several different homogenization techniques can be used. In this paper we have used formulas proposed by Lichtenecker [8], Dobson [9], and Polder and van Santen [10].

The Lichtenecker's formula is expressed by Eq. (6)

$$\lambda_{eff}^{k} = \sum f_j \lambda_j^{k}, \qquad (6)$$

and represents straightforward generalization of Wiener's formula whereas the parameter k varies within the [-1, 1] range.

Because of the large difference between the thermal conductivity of free and bound water in porous medium, Dobson et al. extended the Lichtenecker's power-law formula. They arrived at the following relation

$$\theta = \frac{\lambda_{eff}^{\beta} - \theta_{bw}(\lambda_{bw}^{\beta} - \lambda_{fw}^{\beta}) - (1-\psi)\lambda_s^{\beta} - \psi\lambda_a^{\beta}}{\lambda_{fw}^{\beta} - \lambda_a^{\beta}} \qquad (7)$$

that takes into account the effect of partial water bonding on the pore walls and contribution of thermal conductivity of bound water to the effective thermal conductivity of partially wetted materials. In Eq. (7), θ_{bw} is the amount of water bonded on pore walls [m^3/m^3], λ_{bw} the thermal conductivity of bound water (according to [11], the bound water can be assumed to have the same thermal conductivity as ice, so near -20°C it is equal to 2.4 W/mK), λ_{fw} the thermal conductivity of free water (0.6 W/mK), λ_a the thermal conductivity of air (0.026 W/mK), ψ the total open porosity, and β is an empirical parameter.

Also the effect of the shape of particular constituents can have significant influence on the accuracy and reliability of homogenization formulas for specific groups and types of materials. Polder and van Santen formulated formulas that are valid for spherical inclusions (Eq. (8)) , needle-shape inclusions (Eq. (2)), and board-shape inclusion (Eq (3)). The final formulas can be written in the following way

$$\lambda_{eff} = \lambda_M + \sum f_j (\lambda_j - \lambda_M) \cdot \frac{3\lambda_{eff}}{2\lambda_{eff} + \lambda_j}, \tag{8}$$

$$\lambda_{eff} = \lambda_M + \sum f_j (\lambda_j - \lambda_M) \cdot \frac{5\lambda_{eff} + \lambda_j}{3\lambda_{eff} + 3\lambda_j}, \tag{9}$$

$$\lambda_{eff} = \lambda_M + \sum f_j (\lambda_j - \lambda_M) \cdot \frac{2\lambda_j + \lambda_{eff}}{3\lambda_j}. \tag{10}$$

RESULTS AND DISCCUSION

The results obtained for the reference plaster VO calculated by application of Lichtenecker's formula are presented together with the measured data in Fig. 1. We can see the effect of parameter *k* that was used as empirical fitting parameter for the performed calculations.

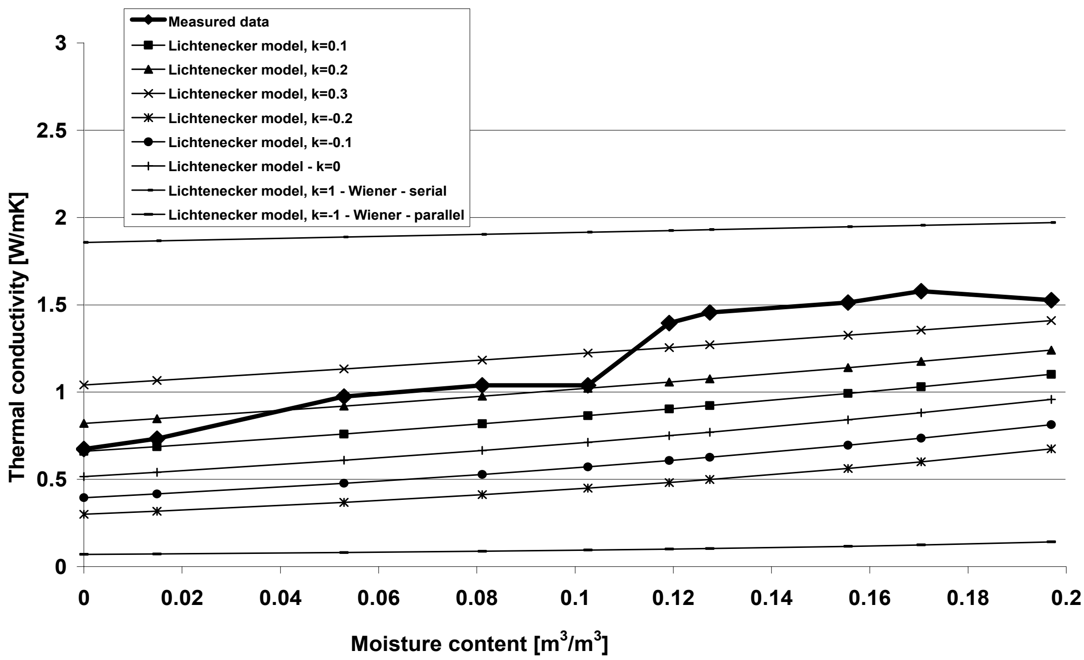

Figure 1 Measured and calculated results for VO, Lichtenecker's formula

Relatively good agreement between calculated results and measured data was obtained in the lower moisture contents, typically up to 10% of volumetric moisture. On the other hand, for the highest moistures, the Lichtenecker's equation completely failed and no reasonable results were obtained. Fig. 2 presents the results obtained by application of Dobson's four phase model for material VO. In this case, based on the parameter β and amount of bound water, the very good agreement between measured and calculated data was obtained in the higher moisture contents.

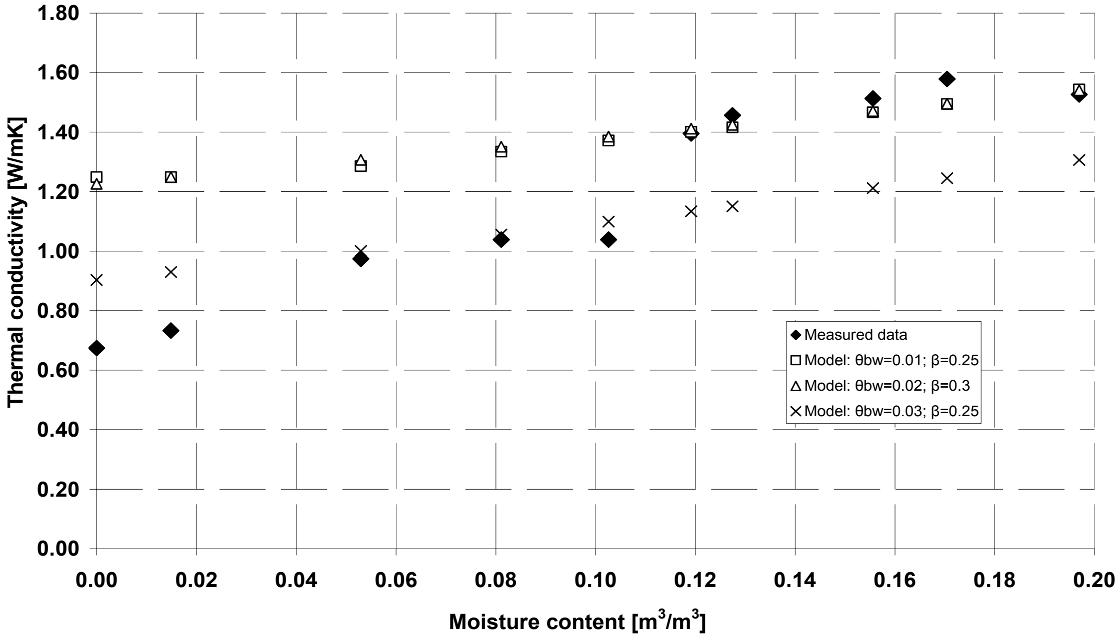

Figure 2 Measured and calculated results for VO, Dobson's formula

Results of application of Polder and van Santen's models for material VO are given in Fig. 3. In this figure, the bounds of effective thermal conductivity function are presented as well.

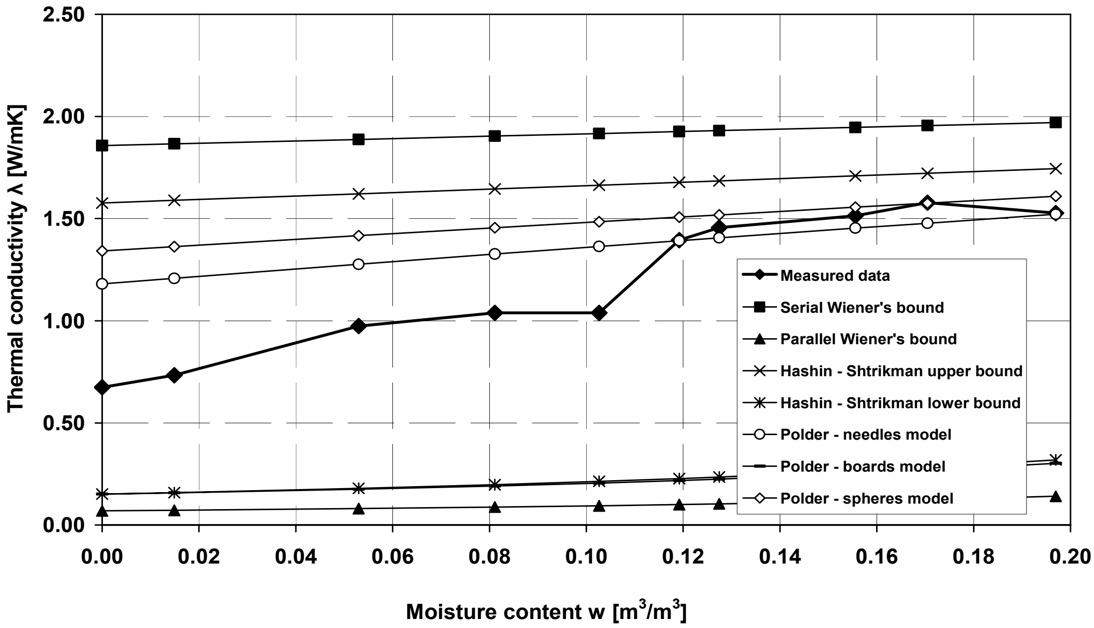

Figure 3 Measured and calculated results for VO, Polder and van Santen's formulas

Very good agreement with measured data was observed again in the range of higher moisture contents but only for model taking into account the needle inclusions in the material. Other tested models completely failed. However, the measured result fulfils the conditions of limiting Wiener's and Hashin-Shtrikman's bounds. This finding can be considered as a certain verification of the performed measurements. The results obtained for material VOM are presented in the following figures.

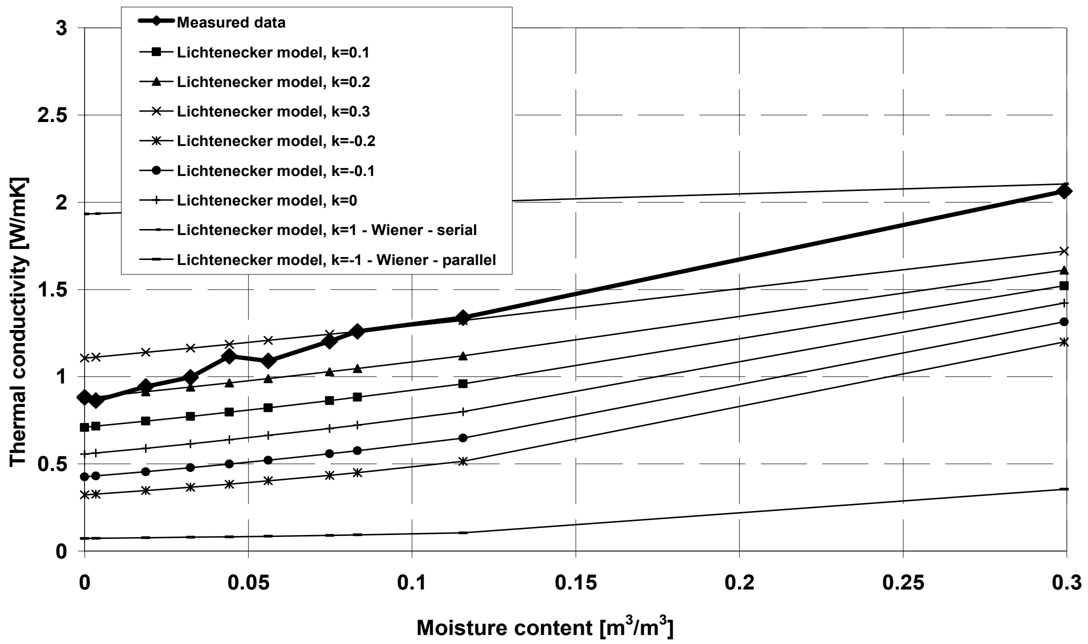

Figure 4 Measured and calculated results for VOM, Lichtenecker's formula

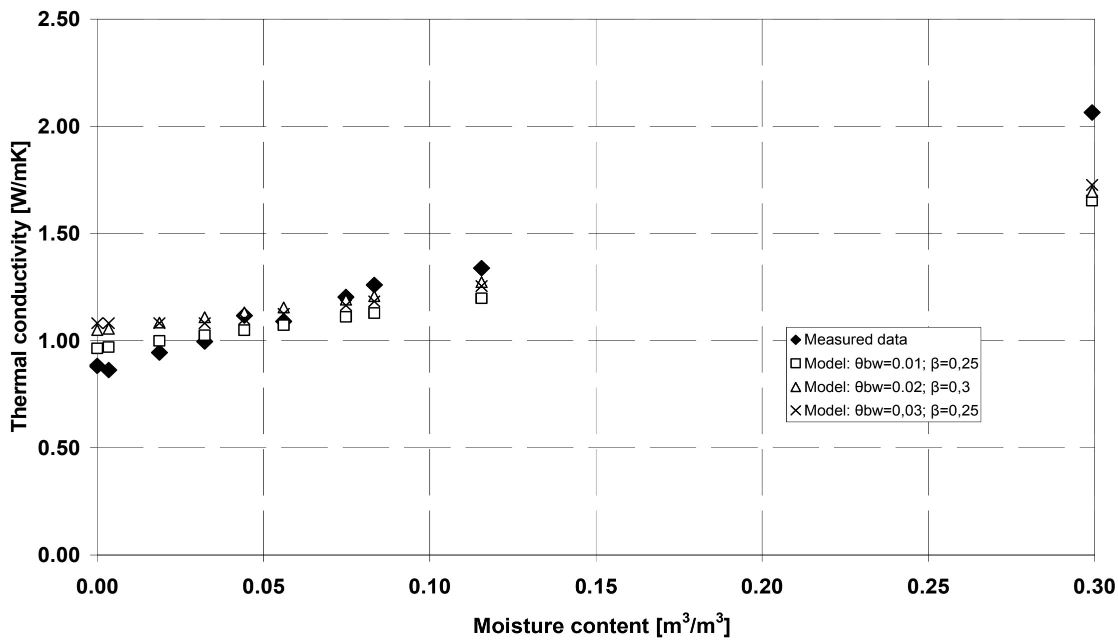

Figure 5 Measured and calculated results for VOM, Dobson's formula

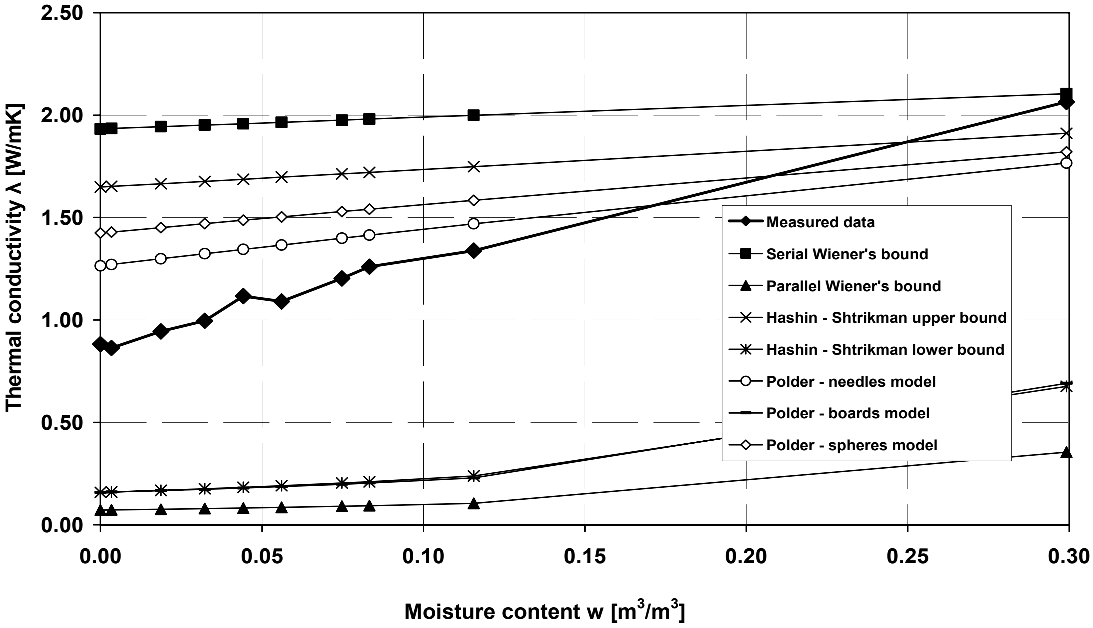

Figure 6 Measured and calculated results for VOM, Polder and van Santen's formulas

In the case of VOM material, the best agreement between measured and calculated data was obtained by application of Dobson's four phase model. However, two fitting parameters were considered in calculations.

CONCLUSIONS

The measured data presented in this paper can find utilization in practical applications of the studied lime based composite renders. The analyzed homogenization techniques were found to be applicable for evaluation of moisture dependent thermal conductivity although the obtained error bar is quit high. Therefore, some other, more detailed analysis will be necessary, with a particular attention to the thermal conductivity values of the components of the lime matrix.

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