Determination of thermal properties from simultaneous monotonic cooling and surface heat transfer measuring

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Abstract:
The monotonic heating regime method for determination of thermal diffusivity is based on the analysis of an unsteady-state (stabilised) thermal process characterised by an independence of the space-time temperature distribution on initial conditions. At the first kind of the monotonic regime a sample of simple geometry is heated / cooled at constant ambient temperature. The determination of thermal diffusivity requires the determination rate of a temperature change and simultaneous determination of the first eigenvalue. According to a characteristic equation the first eigenvalue is a function of the Biot number defined by a surface heat transfer coefficient and thermal conductivity of an analysed material. Knowing the surface heat transfer coefficient and the first eigenvalue the thermal conductivity can be determined. The surface heat transport coefficient during the monotonic regime can be determined by the continuous measurement of long-wave radiation heat flow and the photoelectric measurement of the air refractive index gradient in a boundary layer. The obtained eigenvalues and corresponding surface heat transfer coefficient values enable to determine thermal conductivity of the analysed specimen together with its thermal diffusivity during a monotonic heating regime.

Keywords:
monotonic heating regime, surface heat transfer, thermal properties

1. INTRODUCTION

The method of monotonic heating regime of the first kind, under the boundary conditions of the 3rd kind is suitable for the determination of thermal difussivity of high-density materials [3]. For the determination of complex thermal properties a complementary measurement of the specific heat is necessary [4]. In order to avoid this complication the monotonic heating regime method was combined with a simultaneous measurement of the surface heat tranfer coefficient, especially its convective component. This paper is a modified version of the original publication [5].

2. MONOTONIC HEATING REGIME METHOD

The determination of thermal properties by the monotonic heating regime method of the first kind includes the determination of thermal diffusivity and specific heat of the specimens with a simple geometry. The principle of the measurements is based on the monitoring of a cooling of the specimens with a defined geometry in a constant temperature environment.

Let us consider an isotropic sample of the cubic shape, with a side of length *d*. The initial temperature of the cube is θ_0. At the starting time the cube is suddenly moved from the environment with a constant temperature θ_0 to the environment with a different constant temeperature θ_e ($\theta_0 > \theta_e$). The temperature field of the cube is changing in time and the coefficients of the surface heat transfer are also changing and different at particular cube surfaces during this process. The temperature course at an arbitrary cube point is given by the solution of the heat transfer differential equation (1) for the case of constant material parameters and zero heat sources:

$$a \cdot \nabla^2 \theta = \frac{\partial(\theta)}{\partial t} \tag{1}$$

where: *a* is the thermal diffusivity, θ is the temperature, *t* is the time,
under the boundary conditions of the 3rd kind.

The analysis of the cube cooling under a constant ambient temperature shows that the whole process can be divided into three states:

- At the first state, random, non-steady state the initial temperature distribution is dominant.
- At the second state, known as a monotonic regime, the temperature change with a time runs according to an exponential law.
- The third state corresponds to a steady state, when the temperature in all points of the cube is equal to the ambient temperature.

The second state, the monotonic regime appears after the time of the cooling, when the Fourier number *Fo* is higher than 0.4. Then the time course of relative temperatures of the specimen θ_r at all points under constant boundary conditions can be expressed in the form:

$$\theta_r = \frac{\theta(x,y,z,\tau) - \theta_e}{\theta_0 - \theta_e} = A + A_{x1} \cdot \left(cos\frac{x \cdot \mu_{x1}}{d} + \frac{Bi(d,y,z)}{\mu_{x1}} \cdot sin\frac{x \cdot \mu_{x1}}{d} \right) \cdot A_{y1} \cdot \left(cos\frac{y \cdot \mu_{y1}}{d} + \frac{Bi(x,d,z)}{\mu_{y1}} \cdot sin\frac{y \cdot \mu_{y1}}{d} \right) \cdot A_{z1} \cdot \left(cos\frac{z \cdot \mu_{z1}}{d} + \frac{Bi(x,y,d)}{\mu_{z1}} \cdot sin\frac{z \cdot \mu_{z1}}{d} \right) \cdot exp\left[-\left(\frac{\mu_{x1}^2 + \mu_{y1}^2 + \mu_{z1}^2}{d^2} \right) \cdot a \cdot t \right] \tag{2}$$

where: *Bi(d, y, z)* is Biot number for the surface heat transfer coefficient at the plane parallel the coordinate axes *y, z* at the distance *d*; μ_{x1}, μ_{y1} a μ_{z1} are the first eigenvalues in the directions of the coordinate axes *x, y, z*.

From equation (2) it results that $\ln\theta_r = f(\tau)$ is the line, the tangent of which equals:

$$M = \frac{\partial[ln(\theta_e - \theta(x,y,z,t))]}{\partial\tau} = \frac{ln\theta_r(x,y,z,t_1) - ln\theta_r(x,y,z,t_2)}{t_1 - t_2} = \frac{a}{d^2} \cdot \left(\mu_{x1}^2 + \mu_{y1}^2 + \mu_{z1}^2 \right) \tag{3}$$

The quantity *M* is usually called the cooling rate and it is constant in the monotonic heating regime. The thermal diffusivity is then given by the following relationship:

$$a = \frac{M \cdot d^2}{\mu_{x1}^2 + \mu_{y1}^2 + \mu_{z1}^2} \tag{4}$$

The value *M* can be obtained by calculating the logarithm of the temperature course at an arbitrary point of the cube during the monotonic regime.

The values $\mu_{x1}, \mu_{y1}, \mu_{z1}$ for three pairs of opposite surfaces are the functions of the heat transfer coefficient between given surfaces and the environment. At their determination it is possible to issue from the following assumptions. During the cooling process the cube has surfaces oriented parallel with the coordinate axes, so the heat transfer through 4 vertical surfaces is identical, whilst at the top and bottom horizontal surfaces the heat transfer is different. For the determination of thermal diffusivity then it satisfies to determine only two eigenvalues: $\mu_{x1} = \mu_{y1}$ for vertical surfaces and μ_{z1} for horizontal surfaces of the cube.

At the determination for example of μ_{x1} – value, given by the heat transfer at two opposite surfaces in *x*-axis direction it is possible to issue from the two-points method, based on the fact that the ratio of the temperatures at an arbitrary point of the cube in each moment of the monotonic regime is constant and it is the function of μ_{x1}. Then for the ratios of the temperatures at centres of opposite surfaces to the centre of the cube: k_{xo} and k_{xd} the following relationship is valid:

$$\frac{k_{x0} + k_{xd}}{2} = cos\frac{\mu_{x1}}{2} \tag{5}$$

enabling the calculation of μ_{x1}, or μ_{z1} from the known values of k_{x0} and k_{xd}, or k_{z0} and k_{zd}.

From the cooling rate and the temperatures ratios with use of relations (4) and (5) it is possible to calculate the first eigenvalues μ_{x1}, μ_{z2} and the thermal conductivity of the sample *a*.
The solution of the following transcendent equation enables to obtain the Biot number expressing the relationship between the thermal conductivity of a specimen and the surface heat transfer coefficient:

$$cot\ \mu_{x1} = \frac{\mu_{x1}^2 - Bi_{(0,y,z)} \cdot Bi_{(d,y,z)}}{\mu_{x1} \cdot \left(Bi_{(0,y,z)} + Bi_{(d,y,z)}\right)} \tag{6}$$

As the values of Biot number at opposite vertical surfaces are identical, equation (6) can be simplified.

3. PHOTOELECTRIC MEASUREMENT OF AIR REFRACTIVE INDEX IN BOUNDARY LAYER

The wall surface convective heat transfer coefficient can be defined by the expression combining the empirical Newton law and the steady state Fourier law of heat conduction:

$$h_c = \frac{-\lambda \cdot \left.\frac{d\theta}{dy}\right|_{y=0}}{\theta_{si} - \theta_\infty} \tag{7}$$

where: h_c is the convective surface heat transfer coefficient, $d\theta/dy$ is the air temperature gradient along the vertical *y*-axis, λ is the thermal conductivity of air, θ_{si} and θ_∞ are the temperatures at the surface and outside the boundary layer, respectively.

The experimental determination of the temperature gradient is based on the relationship between the air density dependent on temperature and the air refractive index. This relation is described by the Lorenz-Lorentz law (Fomin 1989):

$$\frac{n^2 - 1}{n^2 + 2} \cdot \frac{1}{\rho} = \frac{N}{M} = const. \tag{8}$$

where: n is the air refractive index [-], ρ is the air density [kg/m^3], N is the air molar refraction [m^3/mol], M is the air molar mass [g/mol].

In the range of 300 – 400 K and under the atmospheric pressure the air can be regarded as an ideal gas. Under the assumption of isobaric condition, the air density change is proportional inversely to the temperature change. Then the refractive index variation is proportional to the air temperature change. A basic constant can be found for the relationship between air temperature change and refractive index change for the wavelength of 650 nm at the temperature of 300 K:

$$\frac{dn}{d\theta} = 0.961.10^{-6} \tag{9}$$

Then, the mutual relationship between the refractive index gradients and temperature gradients is as follows:

$$\frac{d\theta}{dy} = \frac{1}{0.961 \cdot 10^{-6}} \cdot \frac{dn}{dy} \tag{10}$$

Considering the constant distribution of refractive index along the path of optical detection by laser beam and the appropriate geometrical relations, a final equation for the beam deviation in the detector place is:

$$\frac{d\theta}{dy} = \frac{2 \cdot n_0 \cdot \Delta y}{L^2 \cdot 0.961 \cdot 10^{-6}} \tag{11}$$

where: L is the laser beam path length [m], n_0 is the initial refractive index $\cong 1.0$ for air, Δy is the laser beam deviation.

With use of equation (11) the gradient near the surface and then the near-surface temperature profile can be evaluated. Applying then equation (7) the convective heat transfer coefficient can be calculated.

4. EXPERIMENT

The experiment was performed for the polymethyl methacrylate cubic specimen, with the side length of 0.1 m. It consisted of the simultaneous measurement of temperatures at the midpoints of two opposite vertical surfaces and the centre of the cube, the measurement of the laser beam deviations near one vertical surface (2 mm distance) and the measurement of the radiative heat flow between the cube vertical surfaces and the surrounding isothermal surfaces during the cooling experiment. In figures 1 and 2 there are a schematic description of the experiment and a thermograph of the cooled sample. The initial temperature of the specimen was 40 °C and the controlled ambient temperature was 20 °C. The cooling process continued approximately for 3 hours. The monotonic regime lasted during the second hour of the experiment. In figures 3 and 4 there are a time courses of the monitored temperatures and of the near-surface temperature gradient in the air boundary layer calculated by equation (11) from the laser beam deviation data. The corresponding time courses of vertical convective and radiative surface heat transfer coefficients are in figure 5.

An analytical solution of the monotonic cooling of the cube expressed by equation (2) supposes constant boundary conditions. In reality the total surface heat transfer coefficient is decreasing during the proces of cooling. Therefore its average value from the analysed period must be taken into consideration. The time courses of the temperatures ratio k, the first eigenvalue, the total surface heat transfer coefficient, the Biot number and the resultant values of thermal conductivity determined from equations (5) and (6) are in table 1. As the heat transfer at all vertical surfaces of the cube was identical only one *k*-ratio and Biot number could be considered. The average thermal conductivity obtained during the monotonic regime is in agreement with the known value of the polymethyl methacrylate thermal conductivity – 0.185 W/(m.K) determined by the quarded hot plate method.

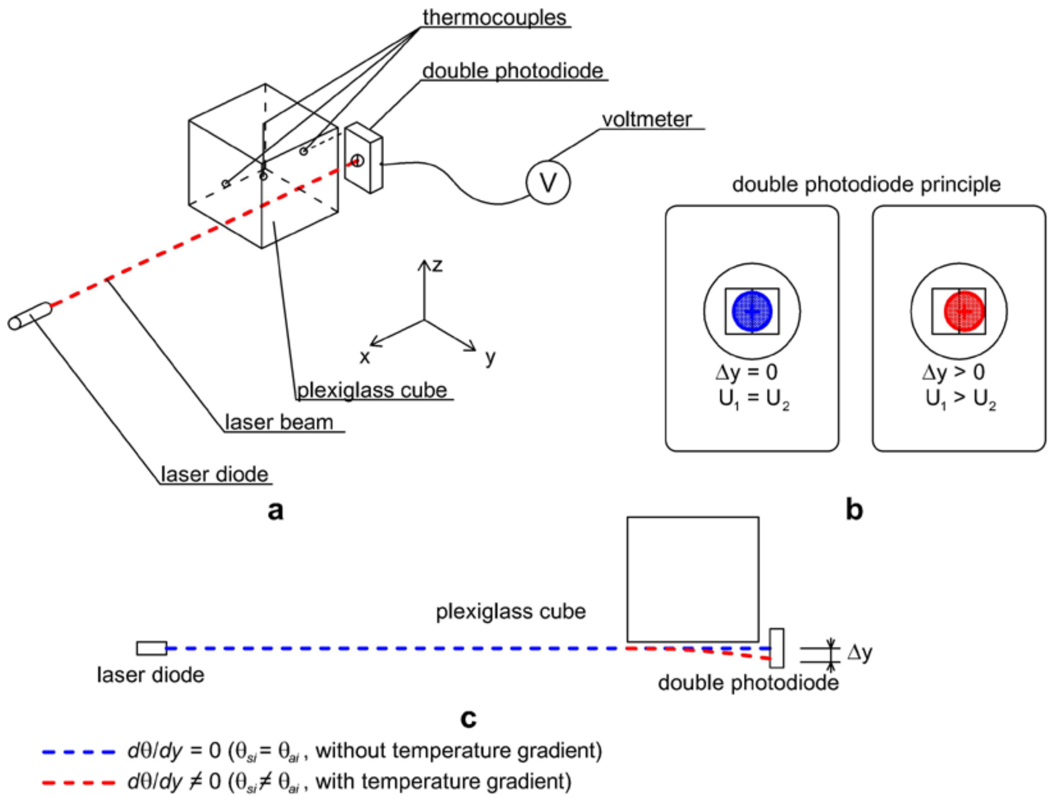

Figure 1 Scheme of experiment

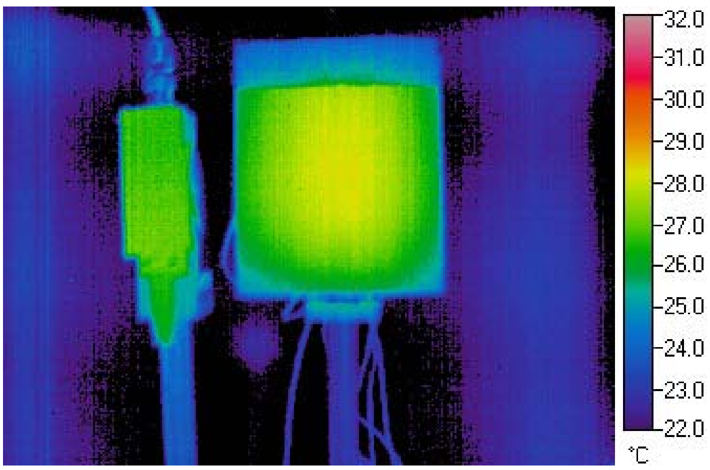

Figure 2 Thermograph of cooled polymethyl methacrylate cube

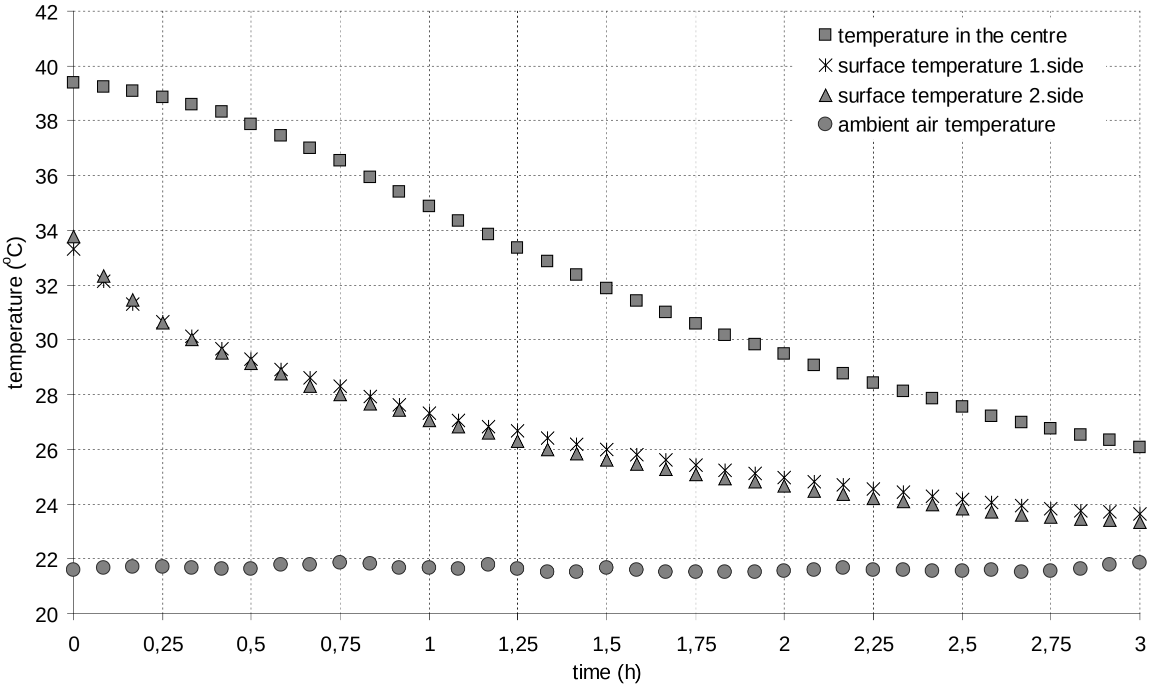

Figure 3 Time course of measured temperatures

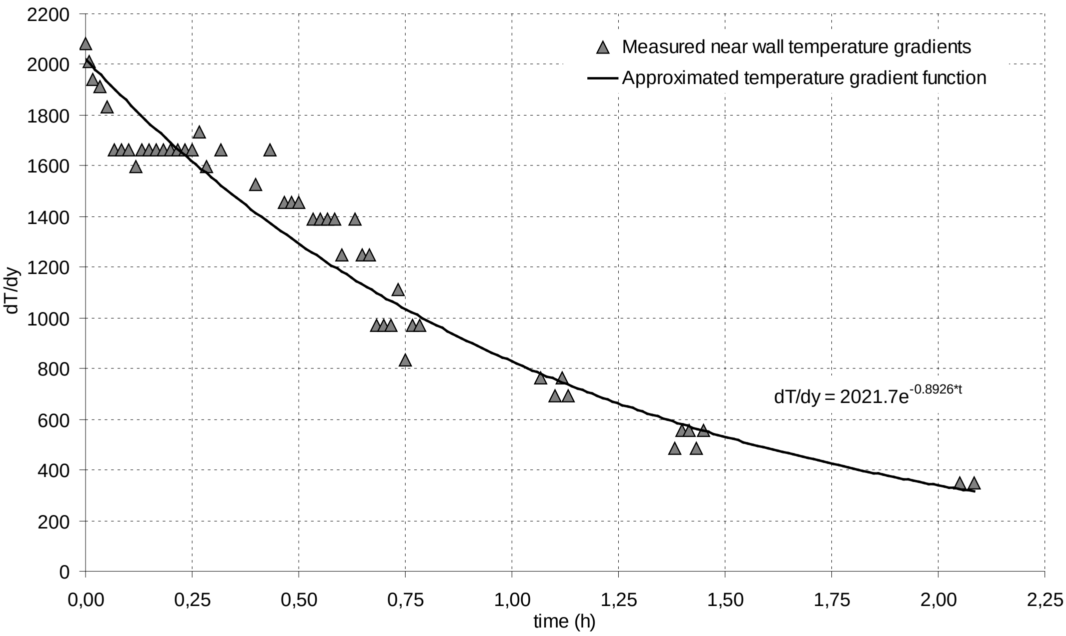

Figure 4 Time course of near-surface temperature gradient

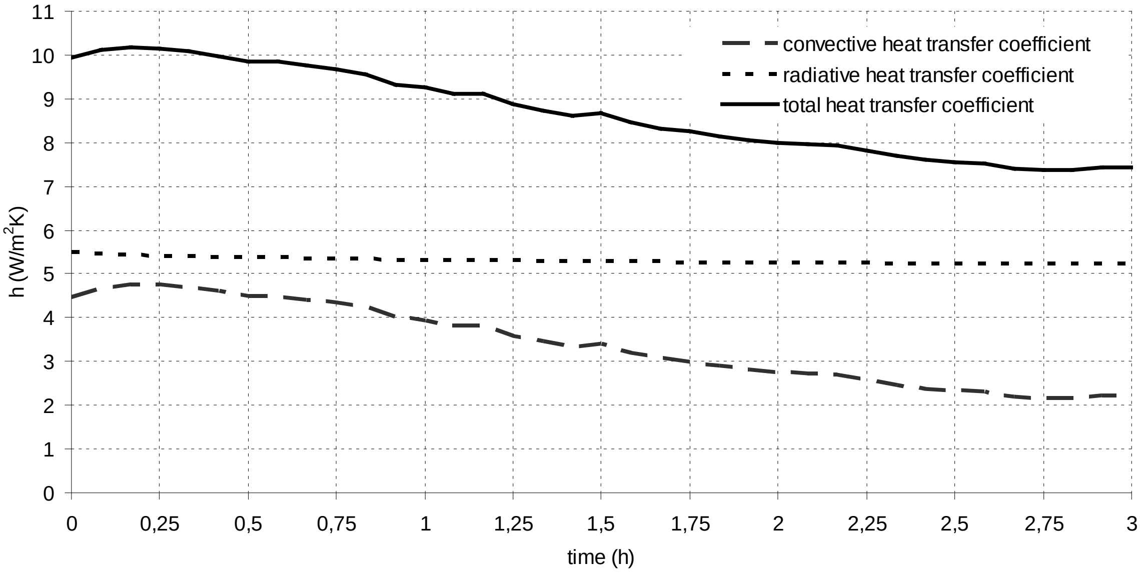

Figure 5 Time course of convective and radiative surface heat transfer coefficients

Table 1 Time courses of measured variables and calculated parameters with resultant thermal conductivities

Time [min]	k [K/K]	μ_1 [1/s$^{1/2}$]	h [W/(m^2.K)]	Bi [-]	λ [W/(m.K)]
60	0.43	2.24	9.25	4.61	0.200
65	0.44	2.24	9.12	4.61	0.198
70	0.44	2.24	9.11	4.66	0.195
75	0.44	2.24	8.87	4.62	0.192
80	0.43	2.25	8.74	4.68	0.187
85	0.43	2.24	8.63	4.67	0.185
90	0.43	2.25	8.66	4.75	0.183
95	0.43	2.24	8.47	4.66	0.182
100	0.44	2.24	8.33	4.63	0.179
105	0.44	2.24	8.24	4.63	0.178
110	0.44	2.24	8.15	4.59	0.177
115	0.44	2.23	8.05	4.57	0.176
120	0.44	2.24	7.99	4.61	0.173
Average	0.44	2.24	8.59	4.64	0.185

5. CONCLUSIONS

An applicability of the simultaneous measurement of the near-surface temperature gradient in air boundary layer during the monotonic heating regime test for the determination of thermal conductivity was proved. The obtained thermal conductivity value was in agreement with the result of independent measurement by the guarded hot plate method.

A combination of the monotonic heating regime method with simultaneous the surface heat transfer measuring enables to determine the thermal diffusivity and the thermal conductivity from one measurement.

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