THEORY AND EXPERIMENTS BY HOT BALL METHOD FOR MEASUREMENT OF THERMAL CONDUCTIVITY

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Abstract

The paper deals with theory and application of a hot ball sensor for measuring thermal conductivity. The sensor, in a form of a small ball, generates heat and simultaneously measures temperature response. A working relation of the sensor has been found on the base of an ideal model. The ideal model represents an empty sphere placed in specimen. Consequently there is a constant heat flow q for $t > 0$ streaming from the empty sphere into surrounding environment. A calibration procedure has been proposed to obtain reliable data. A working range of thermal conductivities of the tested materials has been estimated to be from 0.06 up to 1 W m^{-1} K^{-1}. Ideal model and a model considering heat capacity of the hot ball were tested.

Key words: transient methods, hot ball method, disturbing effects, heat capacity of the hot ball

1 Introduction

Recently a new class of dynamic methods – transient methods for measuring thermophysical properties has started to spread in research laboratories as well as in technology. The technique of transient methods has initiated construction of a range of new innovative laboratory instruments [4-10]. Improvements in methodology of the transient methods and use of recent electronic elements allow construction of some portable instruments and monitoring systems which significantly simplify operation [7, 11]. Construction of such devices has evoked a search for suitable sensors which would provide information on the thermophysical properties of tested objects. Recently a hot ball sensor in a single component configuration, i.e. when a heat source and a thermometer are unified in a single unit, has been presented [13].

The present paper deals with description of basic characteristic of the hot ball technique for the measurement of thermal conductivity. The performance of the sensor has been tested using certified materials. Ideal model and a model considering heat capacity of the hot ball have been tested.

2 Hot Ball Sensor

The working equation of the hot ball sensor is based on an ideal model. The ideal model assumes a constant heat flux F per surface unit from the empty sphere of radius r_b into the infinitive medium starting to be delivered for times $t > 0$. Then the surface of the hot ball is characterized by the function [15]

$$T(r_b, t) = T_0 \left\{1 - \exp\left(u^2\right)\mathrm{erfc}\left(u\right)\right\} \tag{1}$$

where erfc(x) is error function defined by $\mathrm{erfc}(x) = 1 - \frac{2}{\pi}\int_0^x \exp\left(-\zeta^2\right)d\zeta$ and $T_0 = qr_b/\lambda$, $u = \sqrt{at}/\lambda$ and λ, a are thermal conductivity and thermal diffusivity of the surrounding specimen, respectively. The equation (1) is a solution of partial differential equation for heat conduction considering boundary and initial conditions

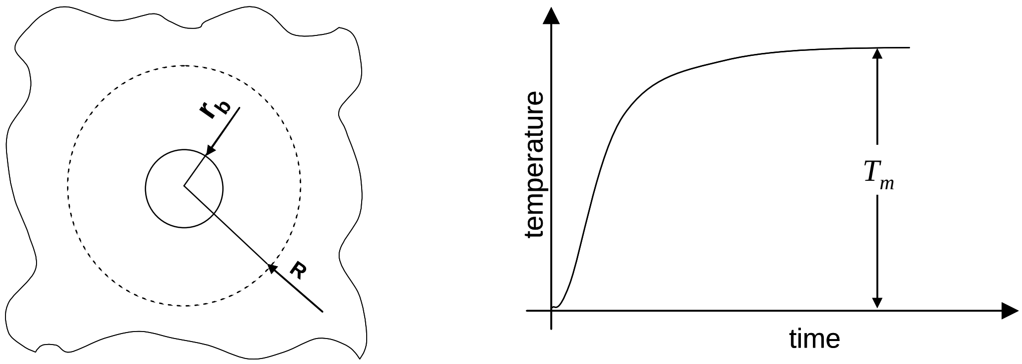


Figure 1. Model of the hot ball (left) and its temperature response for heat output of the ball

$$T(r,0) = 0,$$

$$\lambda \frac{\partial T(r,t)}{\partial r}\Big|_{r=r_b} = -F \text{ , } F = const,\ t > 0.$$

The measuring method based on function (1) belongs, in fact, among the class of transient ones. Nevertheless, the heat source of the spherical symmetry possesses a special feature i.e. it yields the steady state in long times and this moment is utilized to measure the thermal conductivity. It should be stressed that the steady state regime of the hot ball has nothing to do with the one used in the Guarded Hot Plate technique. The latter is based on the existence of the heat and the cold plates (heater and sink) while the former utilizes physics of the heat spread from the spherical heat source. The heat penetrates to sphere with radius R during the temperature stabilization to T_m. Then the determined thermal conductivity corresponds to material within this sphere. Then an averaged value is to be determined for inhomogenous materials. Function (1) gives a working equation (2) of the measuring method in long time approximation $t \to \infty$ assuming that temperature is measured at the surface of the empty sphere r_b

$$\lambda = \frac{q}{4\pi r_b T_m (t \to \infty)} \tag{2}$$

where the heat flux of the empty sphere F is recalculated to the overall ball heat production q according $F = q/4\pi r_b^2$, and T_m is stabilized value of the temperature response. The empty sphere represents an ideal ball of radius r_b characterized by a negligible heat capacity and high thermal conductivity $\lambda_b \to \infty$.

3 Heat capacity of the hot ball

A ball must be constructed of parts generating constant heat on one hand and measuring the temperature response on the other hand. Sensing elements of the hot ball might disturb the measuring process. Therefore an analysis will be performed to estimate the influence of the heat capacity of the hot ball. Then the function that characterises the sensor surface temperature has a form

$$T(R,t) = T_0 \left\{ 1 + \frac{1}{z_1 - z_2} \left[z_2 w\left(= iz_1 \sqrt{t}\right) - z_1 w\left(-iz_2 \sqrt{t}\right) \right] \right\} \tag{3}$$

where

$w(z) = \exp\left(-z^2\right) erfc\left(-iz\right)$, $z_{1,2} = A\left(-1 \pm \sqrt{1-B}\right)$, $T_0 - qr_b / \lambda$, $A = \lambda / \left(2\sqrt{aC_s}\right)$, $B = 4C_s a / \lambda r_b$.

C_s is heat capacity of the unit area of sensor surface and other symbols have been above specified.

4 Calibration of the hot ball

The strategy of the theory verification is based on the calibration of the ball sensors by the Eq. (2) rewritten in a form

$$q / T_m = 4\pi r_b \lambda = A\lambda \tag{4}$$

where A is a constant $A = 4\pi r_b$. The ratio q/T_m is a linear function of thermal conductivity that will be tested using certified materials. A difference between the experimental data and the theoretical function should indicate the weight of the thermal contact and the temperature gradient within the ball. Materials tested within intercomparison measurements by different measurement methods have been used for calibration. The phenolic foam, calcium silicate and PMMA were used for the verification experiments. Table 1 gives thermophysical data of the tested materials together with the structure characterization. The tested specimen consists of two parts and the sensor is placed in the contact of the two specimen surfaces. A groove was made into one part of the specimen in which the ball was placed. A paste (Middland Silicon Ltd) was used for thermal contact improvements. While testing a porous structure the contact surfaces were covered by epoxy varnish to prevent the paste diffusion into material.The ball sensor consists of a heating element and a thermometer. Both elements are fixed in a ball by epoxy resin (Fig. 2 left) (patent pending). Diameter of the ball is in the range of 2÷2.3 mm.

The RTM 1.01 instrument was used for measurements [7]. The scheme of the instrument is shown in Fig. 2 right. A stabilized voltage was used for driving the heating element. Resistance stability of the heater is ±1.5%. Typical measurement signal is shown in Fig. 3 along with the characteristic points used to calculate the thermal conductivity.

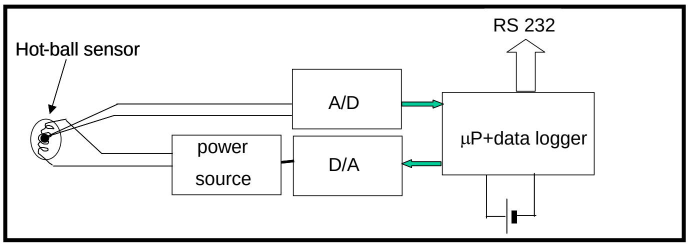


Fig. 2. Photo of the ball sensor (left) and a scheme of the RTM1.01 instrument (right).

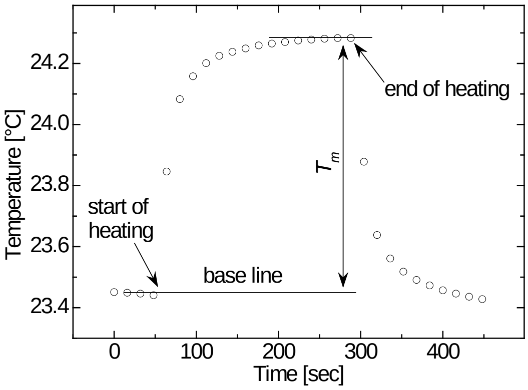


Fig. 3. Measuring cycle. Material: aerated concrete, ball heat output q = 3.5 mW.

The measuring procedure consists of the specimen temperature measurement (base line), switching on the heating and simultaneously scanning the ball temperature. When the ball temperature has stabilized, the heating is interrupted and a period of the temperature equilibration follows. When the temperature in the specimen parts is equilibrated the next measurement may be realized. The repetition rate of the measurements depends on the thermal conductivity and it takes from 10 up to 40 minutes.

5 Results

Evaluation by empty sphere model. A test of the measurement reliability has been performed by measurement of the thermal conductivity of the aerated concrete provided that the ball heat output varies in a broad range. The ball radius r_b = 1.05 mm has been used. Data on thermal conductivity were stable in the range 2.5÷30 mW within $\delta\lambda$ = +/- 0.0007 $W\cdot m^{-1}\cdot K^{-1}$. The measured data are shifted to higher values (λ = 0.265 $W\cdot m^{-1}\cdot K^{-1}$) however the shift is constant within a broad range of ball heat output.

A test of steady-state regime has been performed using the PMMA and measuring parameters q = 0.0025 W, ball radius r_b = 1.05 mm and measuring period (heating time) 3000 s. Equation (2) was used for data evaluation point by point of the scanned temperature response. Data on thermal conductivity started to be stabilized in time window 500 – 3000 s within $\delta\lambda$ = +/- 0.0033 $W\cdot m^{-1}\cdot K^{-1}$. Again data are shifted to higher value λ = 0.265 $W\cdot m^{-1}\cdot K^{-1}$. Data of thermal conductivity obtained on the set of materials specified in Table 1 are shown in Fig. 4. Around 4 measurements of the assembled unit have been performed. At least two reassemblings have been used. A reassembling consists of cleaning the groove, a deposition of the contact paste at the groove point where the hot ball is fixed, fixing the ball into the groove and assembling both parts of the tested materials together into one unit. Analysis of data statistics has shown that the

measurement reproducibility of an assembled unit is rather high. Data scatter is well below 1%. Reassembling induces data scattering within 3-5%.
A theoretical curve is plotted in the Fig. 4 using Eq. (5) where the ball radius is assumed to be r_b = 1.05 mm. A difference between the experimental data and the theoretical curve indicates a disharmony with the ideal model.

Table 1.: Characteristics of materials used for calibration

Material	Thermal conductivity [W m^{-1} K^{-1}]	Structure	Block size [mm^3]
Glycerol	1.9	fluid	-
PMMA	0.19	compact	ϕ50, length 25
Calcium silicate	0.097	porous	150x150x50
Phenolic foam	0.06	porous	150x150x50

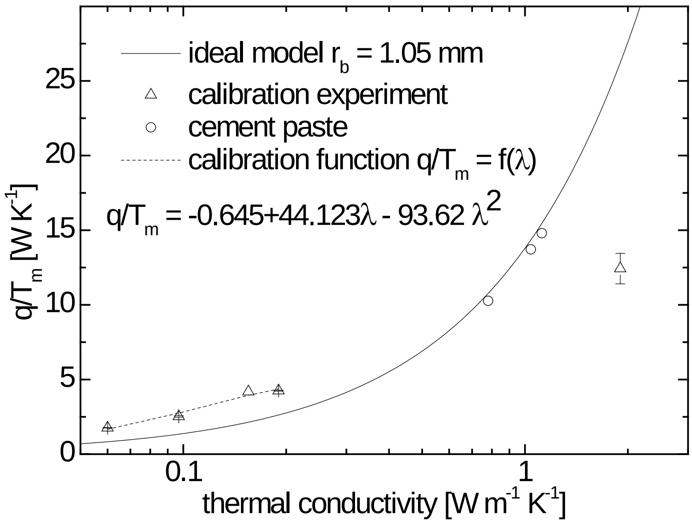
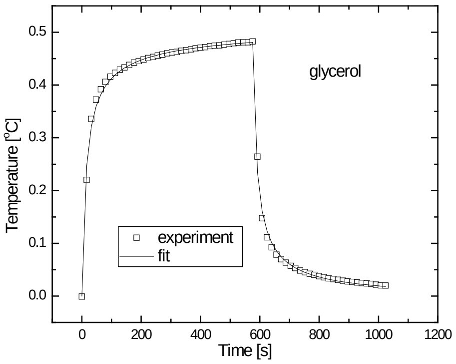


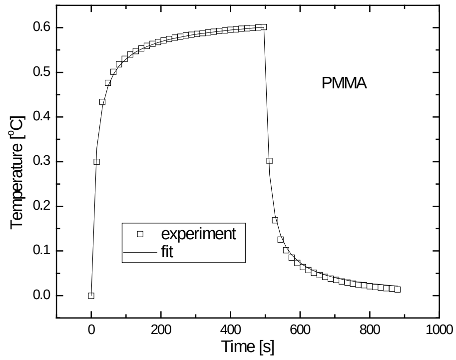
Fig. 4. Calibration of the hot ball sensor.

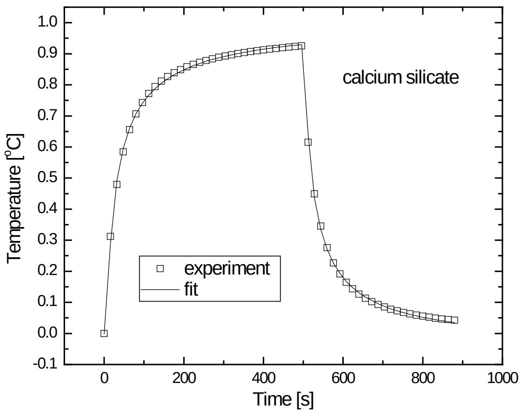
For given q/T_m, a data shift can be found with the low thermal conductivity range and to lower ones for high thermal conductivity range.

Evaluation by heat capacity model. Fitting procedure of Levenberg-Marquat was used for data evaluation using the function (3). Plots in Table 2 show experimental data together with the fitting functions for 4 different materials specified in Table 1. Experimental data were fitted in both in the heating and the cooling period of the hot ball. Four parameters have been fitted, namely thermal conductivity λ, thermal diffusivity a, surface capacity of the hot ball C_s and radius of the hot ball r_b. As too many parameters have to be fitted, the thermal conductivity of the measured materials as an input parameter was used and the radius of the hot ball was used to test the model validity.

Table 2.: Experimental points and fitted function (3).







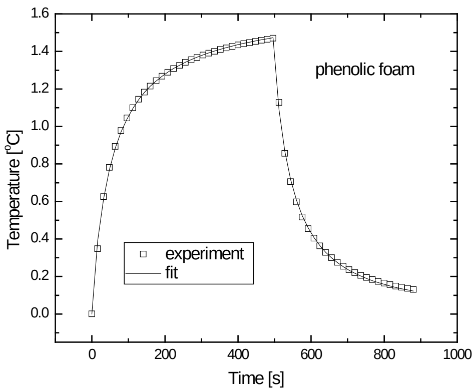


Table 3 gives data on thermal conductivity, thermal diffusivity and the calculated hot ball radius. There is clear discrepancy on hot ball radius considering different materials even fitted function (3) follows close experimental data. This discrepancy indicates that a refined model that includes heat capacity of the hot ball the measuring process is not adequate for characterisation of the measuring process. For a more reliable model an additional disturbing effect has to be included into working relation of the hot ball.

Table 3.: Input and fitted parameters for a range of used materials

Material	Thermal conductivity [W m^{-1} K^{-1}]	Hot ball radius [mm]
Glycerol	0.289	6.49
PMMA	0.19	1.6
Calcium silicate	0.097	2.16
Phenolic foam	0.06	2.3

6 Conclusion

Theory of the hot ball sensor and its experimental verification has been presented. The sensor is based on a ball that generates a constant heat and simultaneously it measures the temperature response for $t > 0$. The ideal model of the empty ball in an infinitive specimen gives a working relation of the hot ball method. The working equation concerns the steady state regime reached after some period of time. The steady state regime could be reached in tens of seconds up to tens of minutes depending on thermal diffusivity of the tested material. Then the thermal conductivity of the specimen is determined by the stabilized value of the temperature response. In addition a revised model considering heat capacity of the hot ball has been presented.
A calibration procedure has based on ideal model been used to analyze the sensor reliability. The experiment has proven that the working range covers the interval from 0.06 up to 0.5 W m^{-1} K^{-1}. A discrepancy has been found considering measured and published data for materials having thermal conductivity in a range from 0.05 up to 0.5 W m K. Data on thermal conductivity are shifted to higher values when real radius of the hot ball was used. A fitting procedure for data evaluation was used using hot ball heat capacity model. A negligible difference between the experimental data and the fitting function was found in both in heating and in cooling period of the hot ball. A radius of the hot ball was extracted from the fitting parameters to intercompare it with the real one. However not realistic data on the hot ball radius were found. This indicates that different disturbing effect is active during measuring process.

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