UNCERTAINTY ASSESSMENT IN EXTENDED DYNAMIC PLANE SOURCE METHOD

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Abstract:
Extended Dynamic Plane Source (EDPS) method can be used to measure simultaneously the thermal conductivity λ and diffusivity a of low thermally conducting materials within a few minutes. However, although the method is relatively simple, the assessment of its uncertainty is a complicated task and *ISO Guide to the Expression of Uncertainty in Measurement* cannot by applied directly. The sources of errors can be divided into three groups. The first group could be defined as the deviation of the experiment from the theoretical model. The second group is caused by random errors. The third group is caused by errors of input parameters measurement. The aim of this contribution is to define the chain of operations required to determine the results and its uncertainty.

Keywords:
standard uncertainty, thermal conductivity, thermal diffusivity, extended dynamic plane source method

INTRODUCTION

The Extended Dynamic Plane Source (EDPS) method [1] is arranged for one-dimensional heat flow into a finite sample. The principle of the method is outlined in Figure 1. The plane source (PS), which simultaneously serves as the heat source and thermometer, is placed between two identical specimens. Heat sink, made of very good heat conducting material, provides isothermal boundary conditions of the experiment. Figure 2 shows the electrical circuit design. Heat is produced by the passage of electrical current through a planar electrical resistance. Turning the switch S on generates the heat flow into both specimens in the form of a step-wise function. Using the constant power resistor, the electrical current and the voltage across PS can be measured. Thus the power, the instantaneous value of PS resistance and temperature can easily be computed.

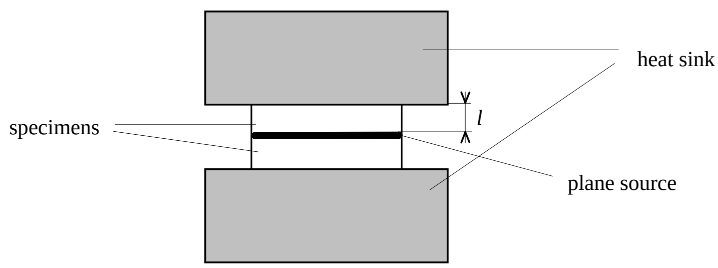

Figure 1. The setup of the experiment.

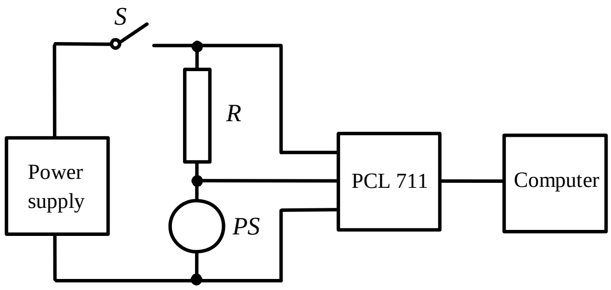

Figure 2. Experimental circuit design. *R* - constant resistor, *S* - switch.

The theoretical model of the experiment is described by the partial differential equation for the heat transport. The temperature function is a solution to this equation with boundary and initial conditions corresponding with the experimental arrangement. The theoretical temperature function is given by

$$T(t,\lambda,a,\tau)=\frac{q}{\lambda}\sqrt{\frac{at}{\pi}}\cdot\left(1+2\sqrt{\pi}\sum_{\mathrm{n}=1}^{\infty}\beta^{\mathrm{n}}\,\mathrm{ierfc}\left(\frac{nl}{\sqrt{at}}\right)\right)+\tau \tag{1}$$

q is the heat current density and *l* is the thickness of the specimen. τ is an additional (nuisance) parameter which represents the offset of PS temperature due to its imperfections. β describes the heat sink imperfection and ierfc is the error function integral [2].

UNCERTAINTY ASSESSMENT

The reliability of every measurement result is confirmed by a quantitative assessment of its uncertainty. General rules for uncertainty assessment have been established in GUM [3]. The sources of error can be divided into three groups. The first could be defined as the deviation of the experiment from the theoretical model.

The model assumes that PS is a homogeneous heat source, has negligible heat capacity and perfect contact with the specimen. These conditions are not exactly fulfilled which causes so called source effect. This is solved by introducing a new parameter τ and removing the beginning of the measured temperature response using difference analysis [4].

It is supposed that there are no heat losses from the lateral sides of the specimen. This can be solved by three methods. The first is removing the end part of the measuring temperature response using difference analysis. The second is measuring with various specimen diameters and extrapolating to infinity diameter. The third is to make the experiment in vacuum.

The model also assumes the constant heat current density i.e. constant electrical power. This is not exactly fulfilled because of the change of PS resistance during the experiment. This can be solved using PC control constant power supply or by measuring with various values of power and extrapolating to zero.

The second group is caused by unknown random errors. These effects can be considered as repeatability of measurement results. Repeatability can be estimated by 10 or more successive measurements carried out under the same conditions and with the same specimen. The apparatus and specimens should be disassembled and reassembled before each measurement. The effect of apparatus assembly is probably one of the most important factors for the results dispersion.

The third group is caused by uncertainties in input parameters measurements. The main sources of uncertainty are connected with the measurement of voltage, resistance of constant resistor, temperature coefficient of resistivity (TCR) of the PS and specimen dimensions.

CONSTANT RESISTOR MEASUREMENT

Measurement of the constant resistor $R \approx 1\ \Omega$ is performed using multimeter M1T 380. Because of low accuracy it could not be done directly but using the following scheme

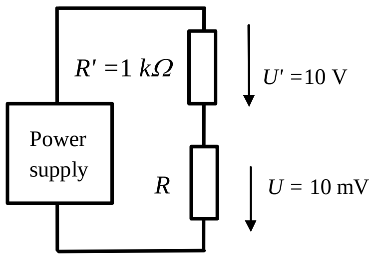

Figure 3. The measurement of the constant resistor R

The value of the resistance of the resistor R is given by the formula

$$R = R'\frac{U}{U'} \tag{2}$$

So three quantities should be measured with errors given by the multimeter producer. Error of $R' \approx 1$ kΩ measurement is 200ppm.MH + 50ppm.MHMR = 275 mΩ. Where MH is the measured value and MHMR is the maximum value of the measuring range. So the standard and relative uncertainties are given by

$$u(R') = \frac{275\ \text{m}\Omega}{1.73} = 160\ \text{m}\Omega \qquad \frac{u(R')}{R'} = 160 \cdot 10^{-6} \tag{3}$$

Error of $U' \approx 10$ V measurement is 100ppm.MH + 20ppm.MHMR = 1.3 mV and the relative standard uncertainty is

$$\frac{u(U')}{U'} = 75 \cdot 10^{-6} \tag{4}$$

Error of $U \approx 10$ mV measurement is 100ppm.MH + 20ppm.MHMR = 4.0 uV and the relative standard uncertainty is

$$\frac{u(U)}{U} = 230 \cdot 10^{-6} \tag{5}$$

Assuming no correlation between input quantities the standard uncertainty of resistance R determination can be computed by root sum square addition as follows

$$\left(\frac{u(R)}{R}\right)^2 = \left(\frac{u(R')}{R'}\right)^2 + \left(\frac{u(U')}{U'}\right)^2 + \left(\frac{u(U)}{U}\right)^2 \tag{6}$$

The standard uncertainty of resistance *R* determination becomes $u(R) = 290\mu\Omega$.

TEMPERATURE COEFFICIENT OF RESISTIVITY TCR MEASUREMENT

The PS was placed into silicon oil bath where the temperature was measured by thermometer with declared expanded uncertainty $U(T) = 0.1\,\mathrm{K}$. The standard uncertainty is given by

$$u(T) = \frac{0.1\,\mathrm{K}}{1.73} = 0.058\,\mathrm{K} \tag{7}$$

The resistance of PS was measured using following schema

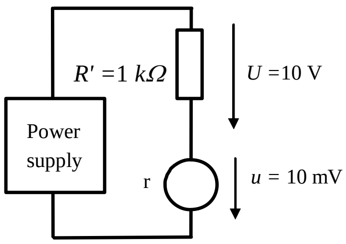

Figure 4. The measurement of PS resistance *r*

and the standard uncertainty of PS resistance *r* ≈ 1Ω determination is as in previous section $u(r) = 290\mu\Omega$.

TCR of PS is defined by the relation

$$r = r_0(1 + \alpha(T - T_0)) \tag{8}$$

where r_0 is the resistance at the temperature T_0. TCR of nickel is $\alpha \approx 0.0047$ / K. The simplest way of determining TCR is to measure temperature and resistance at two points as seen in Figure 5.

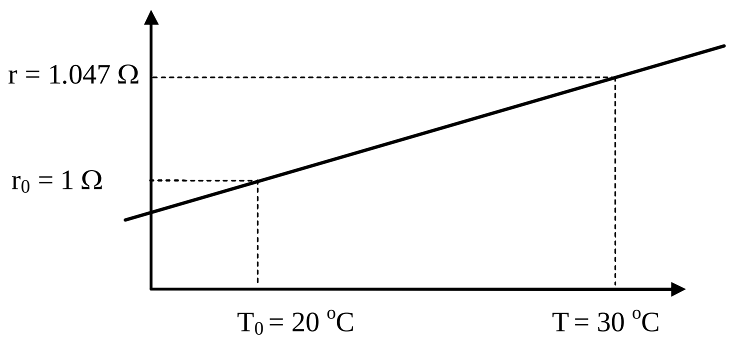

Figure 5. TCR measurement of PS

Then the TCR can be computed using following formula

$$\alpha = \frac{r - r_0}{r_0 \cdot (T - T_0)} \tag{9}$$

The first stage in evaluating uncertainty is to determine the uncertainty of the differences,

$$u(T - T_0) = \sqrt{2} \cdot u(T) = 0.082\ \mathrm{K} \qquad u(r - r_0) = \sqrt{2} \cdot u(r) = 0.41\ \mathrm{m\Omega} \tag{10}$$

then we use the root sum square addition rule

$$\left(\frac{u(\alpha)}{\alpha}\right)^2 = \left(\frac{u(r - r_0)}{r - r_0}\right)^2 + \left(\frac{u(T - T_0)}{T - T_0}\right)^2 + \left(\frac{u(r_0)}{r_0}\right)^2 \tag{11}$$

The standard uncertainty of TCR determination was stated to $u(\alpha) = 56 \cdot 10^{-6}\ \mathrm{K}^{-1}$.

PS RESISTANCE MEASUREMENT

The EDPS experiment consists in measurement of PS temperature response. This is performed by measurement of PS resistance.

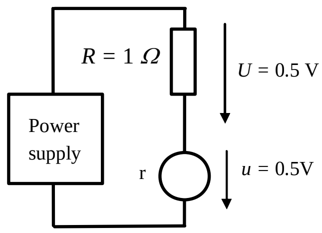

Figure 6. Measurement of time dependence of PS resistance

Now we used current I ≈ 500mA, so the PS is warmed and emitting heat flux. Both voltages *u* and *U* were simultaneously measured using multichannel Advantech PC plug-in card PCL 711. This arrangement removed power supply instability. The declared accuracy by the producer is 0.015 % of reading ± 1 LSB. The quantization noise is suppressed using averaging. The relative standard uncertainty of voltage measurement becomes

$$\frac{u(u)}{u} = \frac{u(U)}{U} = \frac{150 \cdot 10^{-6}}{1.73} = 87 \cdot 10^{-6} \tag{12}$$

The resistance of the PS and its uncertainty are given by the forms

$$r = R\frac{u}{U} \qquad \left(\frac{u(r)}{r}\right)^2 = 2 \cdot \left(\frac{u(u)}{u}\right)^2 + \left(\frac{u(R)}{R}\right)^2 \tag{13}$$

The standard uncertainty of PS resistance determination was stated to $u(r) = 320\mu\Omega$.

HEAT CURRENT DENSITY MEASUREMENT

The heat current density is given by the following forms

$$q = \frac{P}{S} = \frac{U \cdot u}{R \cdot S} \qquad S = \frac{\pi \cdot d^2}{4} \tag{14}$$

uncertainty of PS area *S* determination is given by

$$u(S) = \frac{\partial S}{\partial d} u(d) = \frac{\pi}{2} d \cdot u(d) \tag{15}$$

The specimen diameter was measured using a caliper with a resolution 0.1 mm. The uncertainty becomes $u(d) = 0.1\,\mathrm{mm}/\sqrt{12} = 0.03\,\mathrm{mm}$. Then the relative standard uncertainty of heat current density determination was stated to

$$\frac{u(q)}{q} = \frac{u(S)}{S} = 2\frac{u(d)}{d} = 0.003 \tag{16}$$

THERMOPHYSICAL PARAMETERS ESTIMATION

Inverse problem consists in determining the thermophysical parameters by fitting theoretical temperature function (1) to measured points $[t_i, T_i]$. Since the output of the measurement is the resistance of the PS, the temperature function should be rewritten as

$$r(t) = \frac{\alpha \cdot r_0 \cdot q \cdot l}{\lambda \cdot \sqrt{\pi}} F(t,a) + \rho \qquad F(t,a) = \frac{\sqrt{a \cdot t}}{l}\left(1 + 2 \cdot \sqrt{\pi} \cdot \sum_{n=1}^{\infty} \beta^n \cdot \mathrm{ierfc}\left(\frac{n \cdot l}{\sqrt{a \cdot t}}\right)\right) \tag{17}$$

where α, q ,l are scalar input quantities and $\vec{r}$ is a vector input quantity. ρ is a perturbation parameter stemming from parameter τ. Each input quantity has been determined with its specific uncertainty which contributes to the uncertainty of the thermophysical parameters estimation.

SCALAR INPUT QUANTITIES CONTRIBUTION TO THE COMBINED UNCERTAINTY

The thermophysical parameters are computed using least squares (LS) fitting which can be symbolically expressed as follows

$$y_j = \Phi_j(\vec{r}, \alpha, q, l) \tag{18}$$

where $\vec{y} = (\lambda, a, \rho)$ is a vector of unknown parameters and Φ is an inverse function defined numerically by LS algorithm. According to GUM [3], uncertainty of input quantity *x* contribution to the uncertainty of parameter y_j determination is given

$$u_x(y_j) = \frac{\partial y_j}{\partial x} \cdot u(x) = \frac{\Phi_j(1.01\,x) - \Phi_j(x)}{0.01\,x} \cdot u(x) \tag{19}$$

where the partial derivative is determined numerically.

VECTOR $\vec{r}$ CONTRIBUTION TO THE COMBINED UNCERTAINTY

The result of the measurement is represented by equi-spaced time series of PS resistance r_i ($i = 1 ... n$) denoted as vector $\vec{r}$. Then the sensitivity matrix [5] is given by

$$\{\mathbf{X}\}_{ij} = \beta_j(t_i, \vec{y}) \tag{20}$$

where β_j is the sensitivity coefficient for parameter y_j defined by

$$\beta_j(t, \vec{y}) = \frac{\partial r(t, \vec{y})}{\partial y_j} \tag{21}$$

The standard uncertainty of the LS estimate of the parameter y_j is given by

$$u^2(y_j) = \left\{ \left(\mathbf{X}^T \cdot \mathbf{X} \right)^{-1} \right\}_{jj} \cdot u^2(r) \tag{22}$$

where $u(r)$ is the standard uncertainty of PS resistance estimate.

SUMMARY OF EDPS METHOD

The goal of this work consist in analysing the possible sources of uncertainty in EDPS method. The analysis showed the complexity of uncertainty assessment, though most of operations were simplified or carried out schematically.

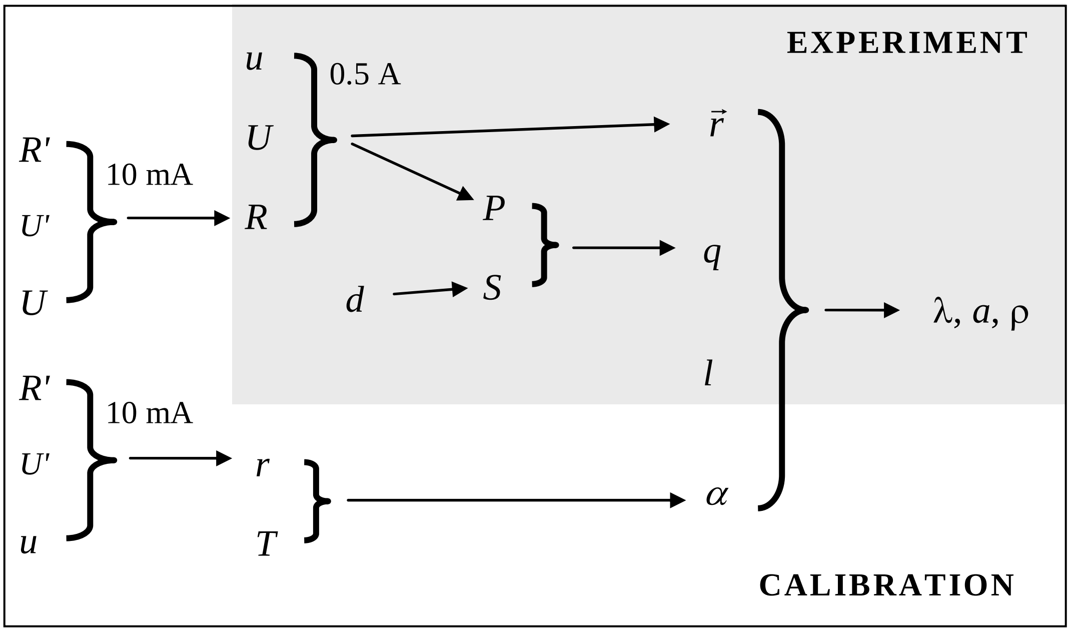

Figure 7. The chain of operations in thermophysical parameters estimation

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