# HOMOGENIZATION TECHNIQUES FOR DETERMINATION OF THERMAL CONDUCTIVITY OF POROUS MATERIALS

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**Abstract:**
Homogenization principles are frequently used for estimation of various parameters of composite materials. Their application is most common in the stress-strain analysis and electromagnetic theory. In this paper, the utilization of mixing rules based on homogenization principles for determination of moisture dependent thermal conductivity is discussed. In terms of homogenization, a porous material is considered as a mixture of three or four phases, namely the solid, liquid and gaseous phase in three-phase systems, and the extra bound-water phase is added in four-phase systems.

**Keywords:**
Thermal conductivity, moisture content, homogenization

## INTRODUCTION

The envelope of any building continuously responds to the changes in indoor and outdoor temperature, air pressure and humidity conditions. This results in an exchange of energy and mass (air as well as moisture) between the indoor and outdoor environments through the envelope. Building physicists refer to these phenomena as “heat, air and moisture transport” through building materials and structures [1]. Designers and builders are always interested, especially for economical reasons and durability and service live problems, in knowing the long-term performance of building envelope, as subjected to the transport processes. That is why the thermal properties of building materials appear to be of particular importance for their practical applications whereas the majority of them contain significant amount of pores that can be in specific cases filled by water. Every catalogue list of any material producer of building materials contains thermal conductivity, sometimes also specific heat capacity but they give only single characteristic values mostly that represent mainly properties of dry materials. In the dry building material heat transfer is a combination of conduction, radiation from the surfaces of the pores and convection within the pores. Thus, in a practical use of thermal conductivity all three modes of heat transfer are to be counted with. However, absolutely dry materials occur in the conditions of building sites very rarely. Also the materials already inbuilt in the structures and exposed to the climatic loading exhibit the dependence of their properties on moisture changes. If the material is wet, heat transferred by moisture in the capillaries and the enthalpy changes that accompany phase transitions also add to the density of heat flow rate. From this appears the necessity to determine thermal conductivity of porous materials as a function of moisture content. Since the experimental measurement of thermal properties in dependence on moisture is very time consuming, new approaches are explored and tested in materials research. In this paper, the applicability of homogenization techniques for determination of moisture dependent thermal conductivity of porous building materials is studied.

## HOMOGENIZATION THEORY AND MIXING FORMULAS

In terms of homogenization theory, the porous material is considered as a mixture of three or four phases, namely the solid, liquid and gaseous phase (in four phase systems, the effect of bound water can be included) that forms the solid matrix and porous space of the material. The solid phase is formed by the materials of the solid matrix. The liquid phase is represented by water and gaseous phase by air. In the case of dry material, only the solid and gaseous phases are considered. The volumetric fraction of air in porous body is given by the measured total open porosity. In case of penetration of water, a part of the porous space is filled by water. For the evaluation of thermal conductivity of the whole material (i.e., the effective thermal conductivity), the thermal conductivities of the particular constituents forming the porous body have to be known.

The effective thermal conductivity of a multi-phase composite cannot exceed the bounds given by the thermal conductivities and volumetric fractions of its constituents. The upper bound is reached in a system consisting of plane-parallel layers disposed along the heat flux vector. The lower bound is reached in a similar system but with the layers perpendicular to the heat flux. These bounds are usually called Wiener's bounds, according to the Wiener's original work [2] and can be expressed by the following relations

$$\lambda_{eff} = \frac{1}{\frac{f_1}{\lambda_1} + \frac{f_2}{\lambda_2} + \frac{f_3}{\lambda_3} + \frac{f_4}{\lambda_4}}, \tag{1}$$

$$\lambda_{eff} = f_1\lambda_1 + f_2\lambda_2 + f_3\lambda_3 + f_4\lambda_4, \tag{2}$$

where Eq. (1) represents the lower limit and Eq. (2) the upper limit of effective thermal conductivity ($f_j$ is the volumetric fraction of the particular phase, $\lambda_j$ its thermal conductivity).

The mixing of phases resulting in effective thermal conductivity functions falling between the Wiener's bounds can be done using many different techniques. We will give couple of characteristic examples of mixing formulas in what follows which were successfully applied by various scientists especially for dielectric mixing in the past and we will introduce them modified for thermal conductivity expressions. Only self-consistent formulas will be accounted for which allow to model the material behaviour in sufficiently wide moisture range.

The Lichtenecker's equation [3]

$$\lambda_{eff}^{\ k} = \sum_{j=1}^{4} f_j \lambda_j^{\,k} \tag{3}$$

is a straightforward generalization of Wiener's formulas. The parameter *k* in Eq. (1) varies within the [-1, 1] range. Thus, the extreme values of *k* correspond to the Wiener's boundary values. The parameter *k* may be considered as describing a transition from the anisotropy at *k* = -1.0 to another anisotropy at *k* = 1.0.

Another mixing treatment was introduced by Rayleigh [4] and a little bit later, with a somewhat different theoretical justification, by Maxwell Garnett [5]. It consists in perception of a continuous phase 1 (in the particular case of a wet porous medium it is the solid matrix) containing randomly distributed spherical scattering particles of discontinuous phases 2, 3 and 4 (in the above mentioned case it is air, free water and bound water, respectively). The formula by Rayleigh can be expressed (in a simple extension from the original 2 to 4 phases) as

$$\frac{\lambda_{eff}-1}{\lambda_{eff}+2}=\sum_{j=1}^{4} f_j\left(\frac{\lambda_j-1}{\lambda_j+2}\right). \tag{4}$$

The formula by Maxwell Garnett (extended to the four-phase system again) can be written as

$$\frac{\lambda_{eff}-\lambda_1}{\lambda_{eff}+\lambda_1}=\sum_{j=2}^{4} f_j\left(\frac{\lambda_j-\lambda_1}{\lambda_j+\lambda_1}\right). \tag{5}$$

The derivation of Maxwell-Garnett's formula is based on the assumption that the basic thermal conductivity of the composite is that of the solid matrix. Bruggeman [6] made a further step towards generalization of this treatment and assumed that the basic thermal conductivity is the thermal conductivity of the mixture. The resulting formula reads

$$\frac{\lambda_{eff}-\lambda_1}{\lambda_{eff}+2\lambda_{eff}}=\sum_{j=2}^{4} f_j\left(\frac{\lambda_j-\lambda_1}{\lambda_j+2\lambda_{eff}}\right). \tag{6}$$

Later, a variety of mixing formulas appeared which reflected the various shapes and topologies of liquid and gaseous phase inclusions within the porous medium. In one of the most popular models of this type Polder and van Santen [7] extended the Bruggeman formula to elliptical inclusions and formulated its three useful simplifications (given in somewhat different algebraic form). The first of them, the original one, is valid for spherical inclusions, the second for needle-shape inclusions and the third for their disc shape. The resulting mixing formulas can be written as

$$\lambda_{eff}=\lambda_1+\sum_{j=2}^{4} f_j(\lambda_j-\lambda_1)\cdot\frac{3\lambda_{eff}}{2\lambda_{eff}+\lambda_j}, \tag{7}$$

$$\lambda_{eff}=\lambda_1+\sum_{j=2}^{4} f_j(\lambda_j-\lambda_1)\cdot\frac{5\lambda_{eff}+\lambda_j}{3\lambda_{eff}+3\lambda_j}, \tag{8}$$

$$\lambda_{eff}=\lambda_1+\sum_{j=2}^{4} f_j(\lambda_j-\lambda_1)\cdot\frac{2\lambda_j+\lambda_{eff}}{3\lambda_j}. \tag{9}$$

Because of the large difference between the thermal conductivity of free and bound water in porous medium, Dobson et al. [8] extended the Lichtenecker's [3] power-law formula and arrived at the relation

$$\theta=\frac{\lambda_{eff}{}^{\alpha}-\theta_{bw}(\lambda_{bw}{}^{\alpha}-\lambda_{fw}{}^{\alpha})-(1-\psi)\lambda_s{}^{\alpha}-\psi\lambda_a{}^{\alpha}}{\lambda_{fw}{}^{\alpha}-\lambda_a{}^{\alpha}}, \tag{10}$$

where $\theta_{bw}$ is the amount of water bonded on pore walls [$m^3/m^3$], $\lambda_{bw}$ the thermal conductivity of bound water (according to [9], the bound water can be assumed to have the same thermal conductivity as ice, so near -20°C it is equal to 2.4 W/mK), $\lambda_{fw}$ the thermal conductivity of free water (0.6 W/mK), $\lambda_a$ the thermal conductivity of air (0.026 W/mK), $\psi$ the total open porosity, and $\alpha$ is an empirical parameter.

De Loor [10] used the Polder-van Santen model [7] for disc inclusions and formulated its extension in the form

$$\theta = \frac{3(\lambda_s - \lambda_{eff}) + 2\theta_{bw}(\lambda_{bw} - \lambda_{fw}) + 2\psi(\lambda_a - \lambda_s)}{\lambda_{eff}(\frac{\lambda_s}{\lambda_{fw}} - \frac{\lambda_s}{\lambda_a}) + 2(\lambda_a - \lambda_{fw})} + \frac{\lambda_{eff}\theta_{bw}(\frac{\lambda_s}{\lambda_{fw}} - \frac{\lambda_s}{\lambda_{bw}}) - \lambda_{eff}\psi(\frac{\lambda_s}{\lambda_a} - 1)}{\lambda_{eff}(\frac{\lambda_s}{\lambda_{fw}} - \frac{\lambda_s}{\lambda_a}) + 2(\lambda_a - \lambda_{fw})}, \qquad (11)$$

where $\lambda_s$ is thermal conductivity of solid matrix.

The introduced mixing models were tested in many practical cases, especially in dielectric mixing applications (see e.g. [11], [12]), and their perspectives for determination of moisture dependent thermal conductivity seem to be very promising.

## CONCLUDING REMARKS

The main objective of the paper was to show the potential for using dielectric mixing models based on homogenization theory for calculation of thermal conductivity of partially water saturated porous building materials. On the basis of previous applications of Bruggeman’s type mixing models and Lichtenecker’s formula for estimation of thermal conductivity of mineral wool boards [13] and cement based composite materials [14] it can be concluded that application of homogenization techniques can provide useful estimates of measured data even for these highly inhomogeneous materials. However, a unified formula could not be found in the whole range of moisture content until now and detailed experimental and theoretical analysis is still needed.

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