APPLICATION OF MAXWELL MIXING RULE IN THE MEASUREMENT OF THERMAL CONDUCTIVITY OF POROUS BUILDING MATERIALS

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Abstract:
Thermal conductivity of a cement-based composite material in dependence on moisture content ranging from the dry state to the water fully saturated state is studied both experimentally and theoretically in the paper. The measurement is performed in laboratory conditions under constant temperature by impulse technique. The experimental data are analysed using the Maxwell-Garnett's mixing rule and the validity of the results is verified by Wiener's bounds. On the basis of performed calculations, the suitability of applied homogenization technique for evaluation of thermal conductivity vs. moisture content function is discussed and its limitations are given.

Keywords:
Thermal conductivity, homogenization techniques, cement-based composites

INTRODUCTION

Cement-based composite materials contain always a significant amount of pores. As the thermal conductivity of the air, filling the porous space of the materials, is approximately 0.026 W/mK [1] and the thermal conductivity of cement stone is (depending on the amount and the type of aggregates) in the range of 1-3 W/mK [2], the total pore volume, distribution of pores, their shapes and cross connections can affect the thermal conductivity of cement-based materials in a very significant way. In usual service conditions of buildings, the cement-based composites always contain certain amount of water that can originate from several sources. The thermal conductivity of water is 0.60 W/mK [1], which is more than 20 times higher than of the air. Therefore, if water is present in the pore space, its effect competes with the effect of air, and the thermal conductivity of a porous composite material can be considered as a result of this competition together with the effect of the cement matrix.

Thermal conductivity as the main parameter describing the heat transport is often subject of measurements for various types of building materials because it plays decisive role in the process of building structures design regarding to thermal resistance and fire protection. In the literature, many examples of thermal conductivity measurements of cement-based materials can be found. However, mostly just one single value is determined (see, e.g., the reviews in [2] – [4]). The dependence of thermal conductivity on moisture content was studied for instance in [5] where empirical relations were obtained but such experiments can be considered as relatively rare.

Homogenization theories working with the concept of an effective medium have proven very useful in a variety of applications in mechanics and in the theory of electricity and magnetism where they already belong to well established treatments (see, e.g., [6]). Their utilization in heat transfer was much less frequent until now. Within the last couple of years, some references appeared on using the effective media theories for estimation of thermal conductivity of refractory materials, foams, polymer-based composites but for cement-based composites their use is still exceptional.

In this paper, experimental determination of moisture dependent thermal conductivity is done for a composite material on cement basis. The experimental data are analysed on the principle of homogenization using the Maxwell-Garnett‘s mixing rule.

EXPERIMENTAL METHOD AND STUDIED MATERIAL

The thermal conductivity was determined using the commercial device ISOMET 2104 (Applied Precision, Ltd.). ISOMET 2104 is a multifunctional instrument for measuring thermal conductivity, thermal diffusivity, and volumetric heat capacity. It is equipped with various types of optional probes, needle probes are for porous, fibrous or soft materials, and surface probes are suitable for hard materials. The measurement is based on the analysis of the temperature response of the analyzed material to heat flow impulses. The heat flow is induced by electrical heating using a resistor heater having a direct thermal contact with the surface of the sample. The measurements were done in dependence on moisture content form the dry state to fully water saturated state. The moisture content in the samples was measured by gravimetric method.

The measurements were done for carbon fiber reinforced cement composite (CFRC) produced in the laboratories of VUSH Brno (CZ). The composition of CFRC material (calculated among the dry substances only) is presented in Table 1. Portland cement used was CEM I 52.5 produced in cement factory Mokrá (CZ), carbon fiber was PAN-type. The water/cement ratio corresponding to the amount of water added into the mixture was 0.9.

Table 1 Composition of the carbon fiber reinforced cement composite in mass-% of dry substances

Cement	Microdorsilite	Plasticizer	Carbon fiber	Wollastonite	Methyl-cellulose	Defoamer	Microsilica
39.71	16.50	0.98	0.98	39.60	0.11	0.16	1.96

The samples were produced using a successive homogenization procedure. First, wollastonite, microdorsilite and microsilica were homogenized in a mixing device, then cement and methylcellulose were added and the dry mixture was homogenized again. The dry well homogenized mixture was thoroughly mixed with water, defoamer and plasticizer. Then, the carbon fibers were added and the mixture shortly mixed again. Finally, the prepared mixture was vacuum-treated in special moulds with perforated bottom. The material was autoclaved at 180 °C and then dried at 105 °C. After the time period of 28 days after mixing, the samples were prepared for testing.

The measured samples were cut from the plates of 10 mm thickness. For any particular moisture content, five specimens of 60 x 60 x 10 mm were used in the measurements of thermal conductivity. The measurements were performed in the laboratory conditions at 24±1 °C and 30-35 % relative humidity.

HOMOGENIZATION THEORY

In terms of a homogenization theory, a porous material can be considered basically as a mixture of three phases, namely solid, liquid and gaseous phase. In the cement based material studied in this work, the solid phase is represented by cement, microdorsilite, carbon fibers and wollastonite, the liquid phase by water and the gaseous phase by air. Therefore, the homogenization was performed in three steps. The first task was the determination of thermal conductivity of the cement matrix. This was done on the basis of the known thermal conductivities and amounts of its constituents. In this work, the thermal conductivity of cement matrix was calculated using the Rayleigh [7] mixing rule

$$\frac{\lambda_M - 1}{\lambda_M + 2} = f_c\left(\frac{\lambda_c - 1}{\lambda_c + 2}\right) + f_m\left(\frac{\lambda_m - 1}{\lambda_m + 2}\right) + f_{cf}\left(\frac{\lambda_{cf} - 1}{\lambda_{cf} + 2}\right) + f_w\left(\frac{\lambda_w - 1}{\lambda_w + 2}\right), \tag{1}$$

where λ_M is the thermal conductivity of cement matrix, λ_c thermal conductivity of cement (2 W/mK), λ_m thermal conductivity of microdorsilite (0.33 W/mK), λ_{cf} thermal conductivity of carbon fibers (9.8 W/mK) and λ_w thermal conductivity of wollastonite (2 W/mK), f_c volumetric fraction of cement, f_m volumetric fraction of microdorsilite, f_{cf} volumetric fraction of carbon fibers and f_w volumetric fraction of wollastonite. The values of thermal conductivities of particular components of cement-based composite were taken from CRC Handbook of Chemistry and Physics [1]. Remaining components forming the solid matrix were not taken into account since their amount in the material is of secondary importance regarding the total thermal conductivity of the studied material.

The second step was the determination of thermal conductivity of the dry material where only the solid and gaseous phases are to be considered. This was realized using the volumetric fraction of the air obtained in porosity measurements and the known thermal conductivities of the matrix and the air. The total open porosity of the studied material was measured by mercury porosimetry and calculated from the total intrusion volume and known bulk density. The results of these measurements are given in Table 2.

Table 2 Basic parameters of the porous space of the studied cement-based composite material

Bulk density [kg/m^3]	Total intrusion volume [cm^3/g]	Total pore area [m^2/g]	Median pore diameter [μm]	Total open porosity [-]
1468	0.216	42.97	0.0236	0.32

For the evaluation of thermal conductivity of the whole material, which is the third and last step of the homogenization procedure, the mixing is performed for cement matrix, air and water. The mixing was done using Maxwell-Garnett’s formula [8] based on the assumption that the basic thermal conductivity of the composite material is that of the solid matrix. The Maxwell-Garnett’s formula extended to the three-phase system (originally it was derived for a two phase system only) is expressed in equation

$$\frac{\lambda_{eff} - \lambda_M}{\lambda_{eff} + 2\lambda_M} = f_a\left(\frac{\varepsilon_a - \lambda_M}{\varepsilon_a + 2\lambda_M}\right) + f_{fw}\left(\frac{\varepsilon_{fw} - \lambda_M}{\varepsilon_{fw} + 2\lambda_M}\right), \tag{2}$$

where λ_{eff} is the effective thermal conductivity of the whole porous material, λ_M the thermal conductivity of cement matrix, λ_a thermal conductivity of air, λ_{fw} thermal conductivity of free water, f_a volumetric fraction of air and f_{fw} volumetric fraction of free water.

For the verification of obtained results, Wiener's bounds [9] for parallel and serial model were used. These bounds in fact represent upper and lower limits of the effective thermal conductivity vs. water content function. The Wiener's bounds are given in the following relations

$$\lambda_{eff} = \frac{1}{\frac{\lambda_M}{\lambda_M} + \frac{f_a}{\lambda_a} + \frac{f_{fw}}{\lambda_{fw}}}, \tag{3}$$

$$\lambda_{eff} = f_1\lambda_1 + f_2\lambda_2 + f_3\lambda_3. \tag{4}$$

RESULTS AND DISCCUSION

The measured results of the dependence of thermal conductivity on moisture content are given in Fig. 1, together with the thermal conductivity vs. moisture content functions calculated using the Maxwell-Garnett's formula and Wiener's parallel and serial bounds.

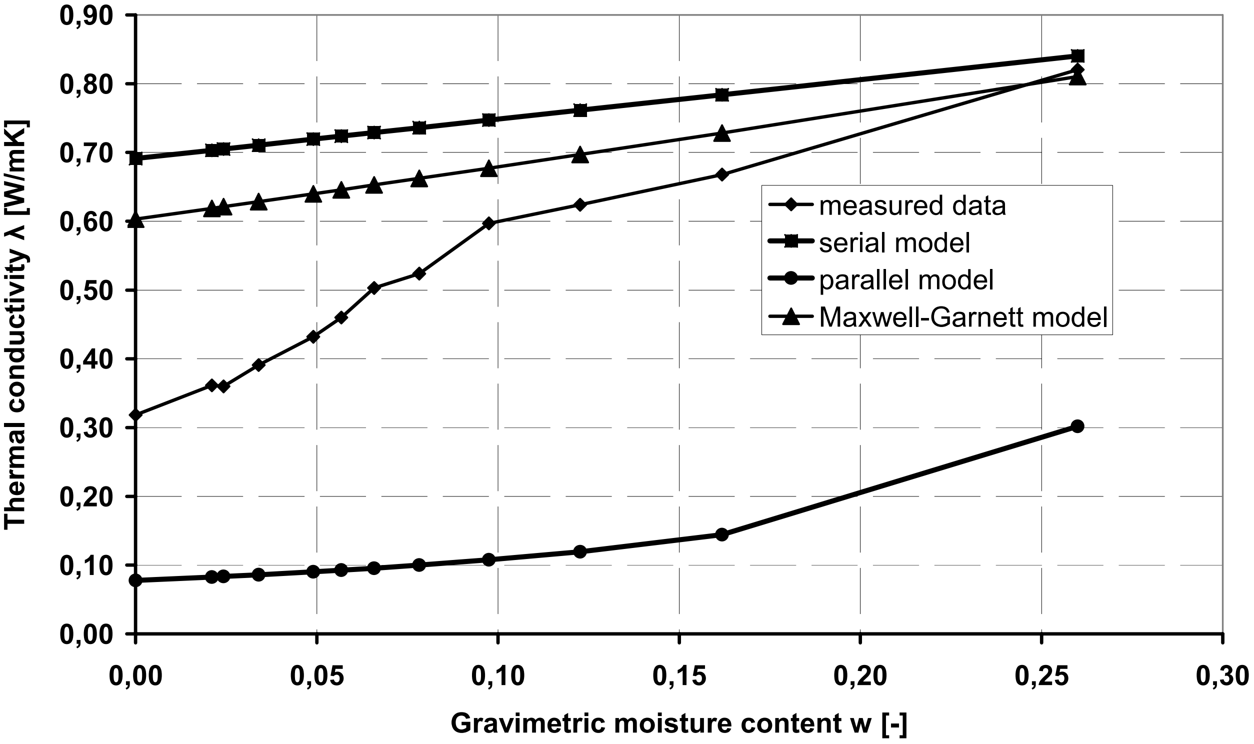

Figure 1 Thermal conductivity of cement based composite material

The experimental results give information on a very substantial moisture effect on thermal conductivity of the studied material which is in basic agreement with its relatively high total open porosity. From the point of view of Wiener's bounds, we can see that the calculated results as well as the experimentally measured data lie between the serial and the parallel model, which basically justifies the reasonable accuracy of both experiment and calculations. Looking at the data from the point of view of accuracy of the analyzed Maxwell-Garnett's formula we can see very high differences (up to 50 %) between measured and calculated data especially for lower moisture contents. For higher moisture contents the measured and calculated values are in slightly better agreement but also here the differences are typically about 10 %. The differences in the range of lower moisture contents are probably caused by the inaccuracies in the thermal conductivity values of the particular components forming the matrix of the material taken from CRC Handbook of Chemistry and Physics [1]. The better agreement in the range of higher moistures can be attributed to the higher effect of water thermal conductivity (measured with sufficient accuracy in many references) on the thermal conductivity of the composite.

CONCLUSIONS

The measured data presented in this paper can find utilization in practical applications of the studied cement based composite material containing carbon fibers. However, the application of Maxwell-Garnett’s mixing formula for the calculation of thermal conductivity in dependence on moisture content was not found to provide useful estimates of measured data. Some other, more detailed analysis will be necessary, with a particular attention to the thermal conductivity values of the components of the cement matrix.

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