INVERSE ALGORITHMS FOR TIME DEPENDENT BOUNDARY RECONSTRUCTION OF MULTIDIMENSIONAL HEAT CONDUCTION MODEL

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Summary: *Comprehensive quantitative information on the heat transfer phenomena is not available for quenching of hot moving surfaces. Attention is focused on the search for boundary conditions describing the heat transfer in engineering applications of spray cooling of metal surfaces. Direct measurements of boundary conditions in many industrial applications or in experiments that simulates these processes are impossible. Thus temperature histories are recorded inside the investigated body and the boundary conditions are computed using inverse heat conduction algorithms using experimental data. Sequential Beck's inverse algorithm and identification methods are discussed. Combining measurement with an inverse analysis often results in an ill-posed problem. Such problems are extremely sensitive to measurement errors. The distance of the measurement point from the investigated surface strongly influences the shortest impulse that can be reconstructed by an inverse method. Based on magnitude of stabilization factor the degradations of reconstructed boundary conditions are presented..*

1. Introduction

For computational methods knowledge of boundary conditions is necessary. Those conditions can be computed for simple cases, however, they must be obtained from measurements in most cases. Boundary conditions can be measured directly on the surface or if not possible we can do the measurement inside the investigated body and then we have to use an inverse task to compute boundary conditions from measured values. In our case we concentrate on boundary conditions during water cooling of hot steel products or of hot working rolls (Horský 2005). In these cases it is not possible to measure cooling intensity directly on the surface and we have to measure temperature history inside the body and to compute boundary conditions using an ill-posed inverse task. The accuracy of the computed results is strongly dependent on two factors: distance of the thermocouple from the investigated surface and on the additional noise in the measured data.

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2. Measurement

Experimental conditions are prepared in such way, which resembles as close as possible to the real mill conditions (Raudenský 2003). There are two basic parameters, which should be kept. The first is the initial temperature of tested sample and the second is the speed of sample motion. To measure boundary conditions a special experimental stand was developed for these tests.

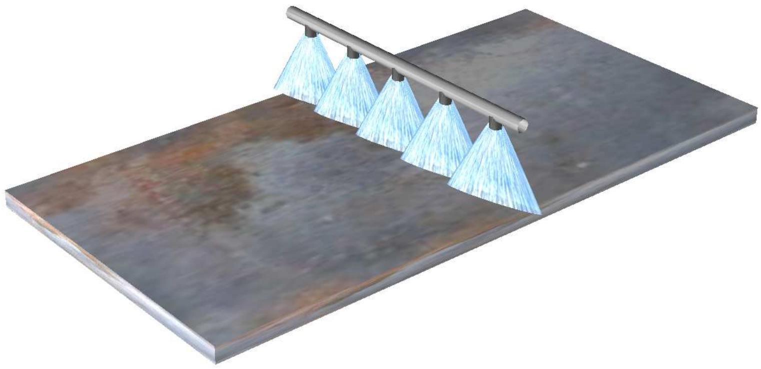

Figure 1 – High-pressure water nozzles with flat water stream removing oxide layers from a hot surface steel.

Experimental stand

The experimental stand was built to study the cooling of linearly moving objects. A six meter long girder carrying a movable trolley and a driving mechanism (see Figure 2) forms the basic part of the experimental device. An electronic device measuring the instant position of the trolley is embedded in the trolley. The driving mechanism consists of an electric motor controlled by a programmable unit, a gearbox, two rollers and a hauling rope. The girder is divided into three sections. The marginal sections are used for the trolley's acceleration or deceleration. The velocity of the trolley is constant in the mid-section and it is here where the spray nozzles quench the measured sample.

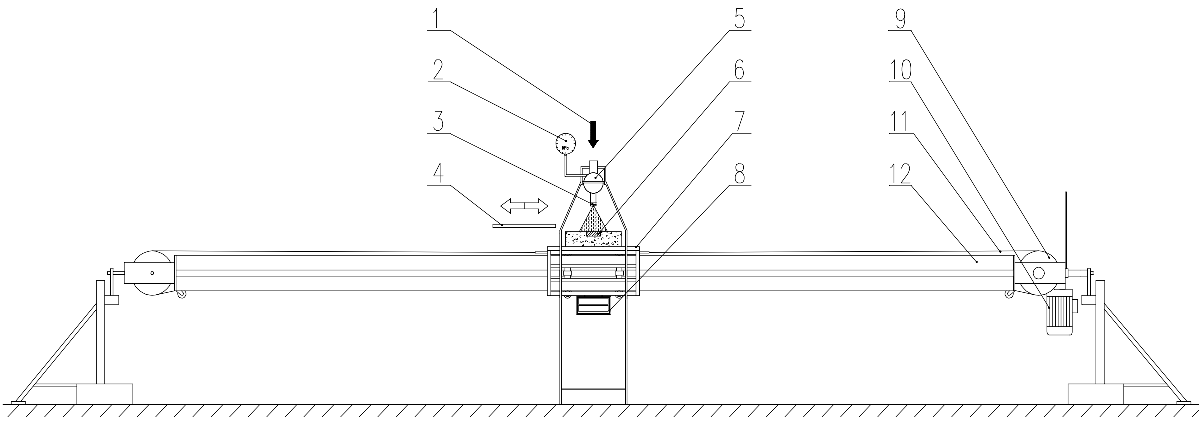

Figure 2 – Principal scheme of the linear test bench (1-cooling medium supply, 2-pressure gauge, 3-nozzle, 4-moving deflector, 5-manifold, 6-tested sample, 7-moving trolley, 8-datalogger, 9-roller, 10-electric motor, 11-hauling steel wire rope, 12-girder).

The procedure of the experiment is as follows:

- An electric heater heats the test plate to an initial temperature of the experiment.
- The plunger water pump is switched on and the pressure is adjusted.
- A driving mechanism moves the test plate under the spraying nozzles. After recovering the temperature field in the plate, the movement of the plate under the spraying nozzles is repeated.
- The temperature is measured using special temperature sensor inside the investigated steel plate and the temperature is recorded into data logger memory.
- The positions of the test plate and the thermocouples (in the direction of movement) are recorded together with the temperature values. The record of instant positions is used for computation of instant velocities and positions while moving under the spray.

3. Sensor description

To measure temperature inside the body, special sensors with built-in K-thermocouples are used as shown in Figure 3. The main body of the sensor is made of stainless austenitic steel. A hole of 1.1 mm in diameter for a thermocouple is made from the side of the sensor. The axis of the hole is 1 mm under the investigated surface and is perpendicular to the expected heat flux, so that the most important part of the inserted thermocouple lies in one isotherm. Inside the sensor, a shielded ungrounded K thermocouple is placed. The gap between the sensor and the thermocouple is filled with copper or ceramic material that can be exposed to a higher temperature than copper.

Sensors of this type are used mainly for descaling experiments but very similar sensors are used for measuring temperature during the rolling process. During measurements that serve for computing heat transfer coefficient (HTC), the sensors are placed in the steel object on which the HTC is investigated (see Figure 1).

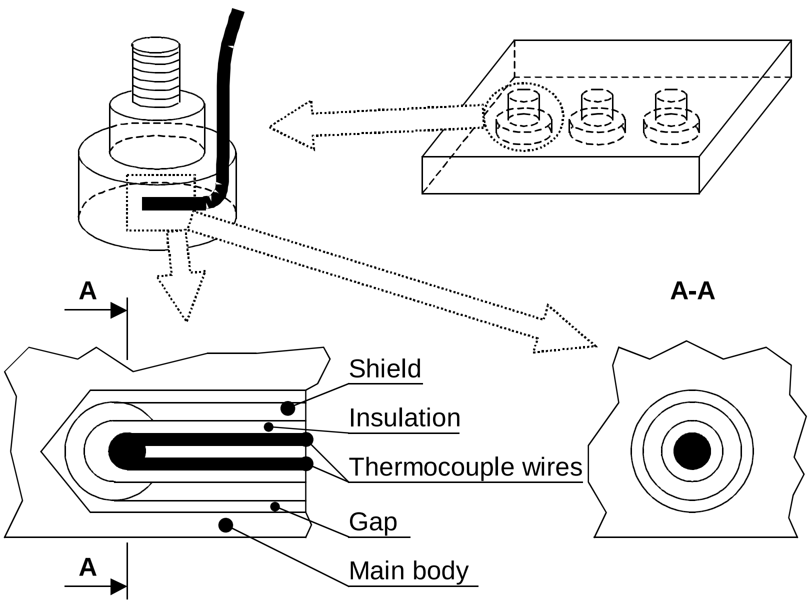

Figure 3 – Application of sensor, and its structure in detail.

None sensors are exactly the same. The positions of the junction point inside the shielded thermocouple differ. Also the hole inside the sensor is a bit bigger than the thermocouple so that its position can differ. As the thickness of the material in the gap differs, the heat resistance does too. These are the main reasons why the calibration experiment are done for each sensor (Pohanka 2002). An optimization method is used for finding the appropriate thermal conductivity of the material that fills the gap in the upper and lower parts and depth of installed thermocouple.

4. Computational Model

A 2D axis symmetric model was used as shown in Figure 4. The model includes the shielded thermocouple with all its parts. The thermocouple must be taken into account because the homogeneity of material is disturbed by the inserted thermocouple, and thus the temperature profile is also disturbed.

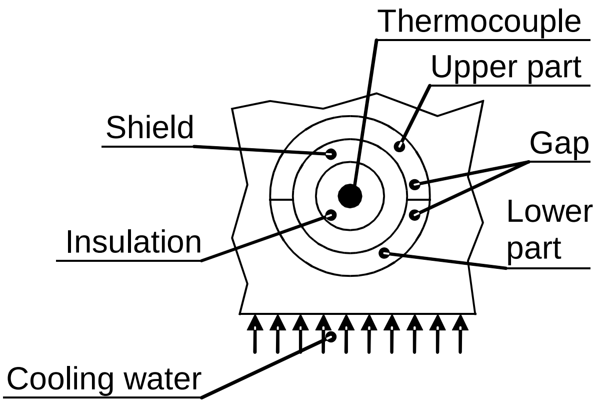

Figure 4 – Computational 2D model.

An example of the temperature field around the thermocouple is shown in Figure 5. A quite flat circular part represents the cross-section of the thermocouple. The surface temperature (at Y=0 mm) is also disturbed by the installed thermocouple.

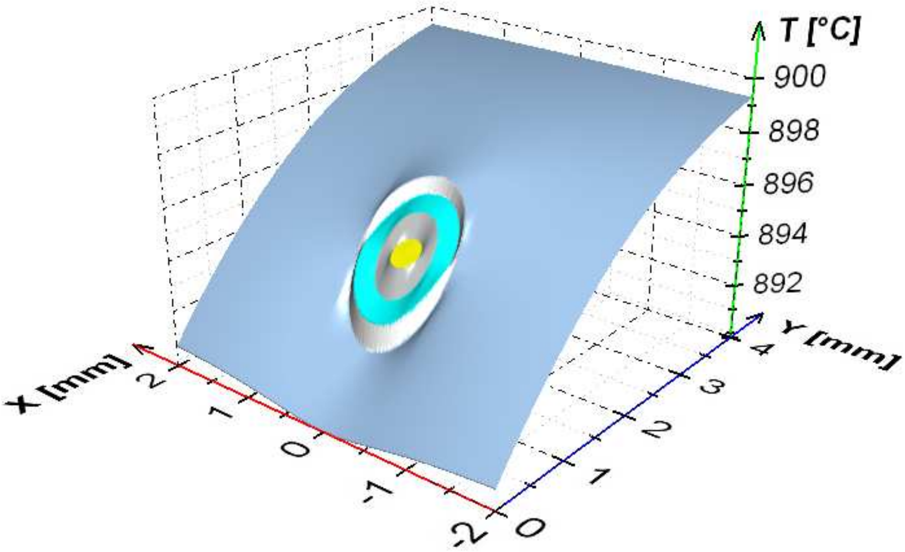

Figure 5 – Temperature profile of the optimized 2D model.

This 2D model and the general unsteady heat conduction equation

$$\frac{\partial}{\partial x}(k\frac{\partial T}{\partial x})+\frac{\partial}{\partial y}(k\frac{\partial T}{\partial y})=\rho\cdot c_p\frac{\partial T}{\partial t} \quad (1)$$

are used for computing the temperature profile and temperature history Incropera (1996). The Control Volume method is used for solving Eq. (1) as described in Patankar (1980). This 2D model is fully insulated on the surface except the investigated surface with water cooling.

5. Evaluation using inverse task

The pass under the nozzle causes temperature drop in the material sample. This information together with material properties and calibration characteristics of temperature sensor is used as an input for the inverse heat conduction task. The results of computation are surface temperature, heat flux and heat transfer coefficient (HTC). Two approaches will be discussed here: sequential Beck's approach (1985) and Identification method Raudenský (2002).

Sequential Beck's approach

The main feature of Beck's approach is sequential estimation of the time varying boundary conditions. Beck demonstrated that function specification and regularization methods could be implemented in a sequential manner and that they gave in some cases nearly the same results as the whole domain estimation. Moreover the sequential approach is computationally more efficient. Beck's approach has been widely used to solve inverse heat conduction problems to determine unknown boundary or material property information.

The method uses sequential estimation of the time varying boundary conditions and uses future time steps data to stabilize the ill-posed problem. The HTC is found after determining the heat flux at the surface. To determine the unknown surface heat flux at the current time t^m, the measured temperature responses $T_i^{*,m}$, are compared with the computed T_j^m from the forward solver (e.g. FDM, FVM, FEM, etc.) (Patankar 1980), using n_f future time steps

$$SSE=\sum_{f=m+1}^{m+n_f}\sum_{j\equiv i;i=1}^{n_{T^*}}(T_i^{*,f}-T_j^f)^2 . \quad (2)$$

Using the linear minimization theory, the value of the surface heat flux that minimizes Eq. (2) is

$$\hat{q}^m=\frac{\sum_{f=m+1}^{m+n_f}\sum_{j\equiv i;i=1}^{n_{T^*}}(T_i^{*,f}-T_j^f|_{q^m=0})\zeta_i^f}{\sum_{f=m+1}^{m+n_f}\sum_{i=1}^{n_T}(\zeta_i^f)^2} \quad (3)$$

where $T_j^f|_{q^m=0}$ are the temperatures at the temperature sensor locations computed from the forward solver using all the previously computed heat fluxes, but without the current one q^m. The ζ_i^f is the sensitivity of the i^{th} temperature sensor at time t^f to the heat flux pulse at time t^m. These sensitivity coefficients are mathematically the partial derivatives of the computed temperature field to the heat flux pulse, but in this case they physically represent the rise in temperature at the temperature sensor location for a unit heat flux at the surface. The sensitivity coefficient of our interest is defined as

$$\zeta_j^m = \frac{\partial T_j^m}{\partial q^m} \tag{4}$$

Once the heat flux is found for the time t^m, the corresponding surface temperature T_0^m may be computed using the forward solver. When the surface heat flux q^m and surface temperature T_0^m are known, the heat transfer coefficient is computed from

$$h^m = \frac{\hat{q}^m}{T_\infty^m - (T_0^m + T_0^{m-1})/2} \,. \tag{5}$$

This approach is limited to linear problems. However, it can be extended to nonlinear cases. The modification of this procedure involves an outer iteration loop which continues until the computed temperature field is unchanging. The nonlinearity requires iteration only to determine the present value of the heat flux, while the computations to determine the surface temperature and heat transfer coefficient need only be performed once for each time t^m. The sensitivity coefficients are also nonlinear, due to the dependence of the thermal properties on the temperature field, and they must be computed for each iteration.

Once the heat transfer coefficient at the "present" time is computed, the time index *m* is incremented by one, and the procedure is repeated for the next time step. For *n* measured time steps only *n* – *f* can be computed owing to the use of future data as a regularizing approach.

Sequential Beck's approach in multi-dimensions

The sequential approach can also be used for multidimensional IHCP. The temperatures in one-, two-, or three-dimensional objects with temperature independent thermal properties can be obtained using

$$\boldsymbol{T} = \boldsymbol{T}|_{\boldsymbol{q}=\boldsymbol{0}} + \varsigma \boldsymbol{q} \tag{6}$$

where

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{T}(m) \\ \boldsymbol{T}(m+1) \\ \vdots \\ \boldsymbol{T}(m+f-1) \end{bmatrix}, \quad \boldsymbol{T}(i) = \begin{bmatrix} T_1^i \\ T_2^i \\ \vdots \\ T_{n_{T^*}}^i \end{bmatrix}, \quad \boldsymbol{q} = \begin{bmatrix} \boldsymbol{q}^{(m)} \\ \boldsymbol{q}^{(m+1)} \\ \vdots \\ \boldsymbol{q}(m+f-1) \end{bmatrix}, \quad \boldsymbol{q}(i) = \begin{bmatrix} q_1^i \\ q_2^i \\ \vdots \\ q_{n_{q^*}}^i \end{bmatrix},$$

$$\varsigma = \begin{bmatrix} \boldsymbol{\zeta}(1) & & & \\ \boldsymbol{\zeta}(2) & \boldsymbol{\zeta}(1) & & \\ \vdots & & \ddots & \\ \boldsymbol{\zeta}(f) & \boldsymbol{\zeta}(f-1) & \cdots & \boldsymbol{\zeta}(1) \end{bmatrix}, \quad \boldsymbol{\zeta}(i) = \begin{bmatrix} \zeta_{1,1}(i) & \cdots & \zeta_{1,n_{q^*}}(i) \\ \vdots & \ddots & \\ \zeta_{n_{T^*},1}(i) & & \zeta_{n_{T^*},n_{q^*}}(i) \end{bmatrix} \tag{7}$$

The sequential approach then temporary assumes that q is independent on time. Then using

$$\boldsymbol{Z} = \varsigma \boldsymbol{I}^* \quad \text{where} \quad \boldsymbol{I}^* = \begin{bmatrix} 1_1 & & 0 \\ & \ddots & \\ 0 & & 1_n \end{bmatrix} \quad \text{where} \quad n = n_{q^*} \tag{8}$$

the function to minimize is

$$SSE = (\boldsymbol{T}^{*,m} - \boldsymbol{T}^m|_{\boldsymbol{q}=0} - \boldsymbol{Z}^m \boldsymbol{q}^m)^T (\boldsymbol{T}^{*,m} - \boldsymbol{T}^m|_{\boldsymbol{q}=0} - \boldsymbol{Z}^m \boldsymbol{q}^m) \tag{9}$$

The matrix derivative of Eq. (9) with respect to $\boldsymbol{q}$ gives the estimated heat fluxes

$$\hat{\boldsymbol{q}}^m = [(\boldsymbol{Z}^m)^T \boldsymbol{Z}^m]^{-1} (\boldsymbol{Z}^m)^T (\boldsymbol{T}^{*,m} - \boldsymbol{T}^m|_{\boldsymbol{q}=0}) \, . \tag{10}$$

After it is obtained, m is increased by one and the procedure is repeated for the next time step.

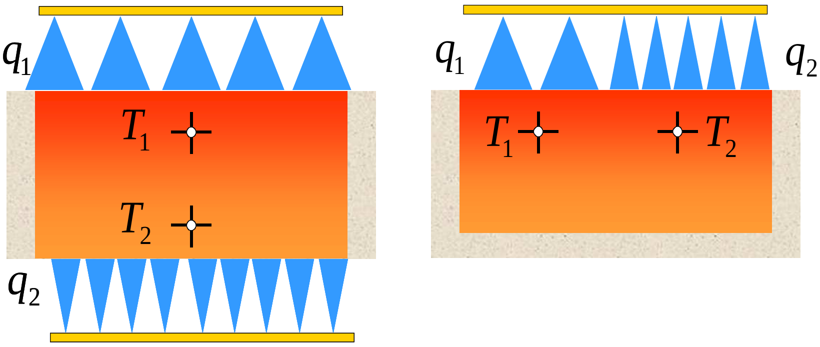

Figure 6 – Multidimensional models and multiple heat fluxes.

Identification method

Probably the main disadvantage of the Beck algorithm is that it assumes constant heat flux (in space domain) on the surface close to the installed thermocouple. As we saw in Figure 5, the surface temperature is disturbed by the installed thermocouple. Knowing that HTC is constant for wide temperature ranges we find out using Eq. (5) that the surface heat flux is not constant. It is even more obvious for surface temperatures approaching temperature of the coolant medium. The disturbances in our cases were over 30%.

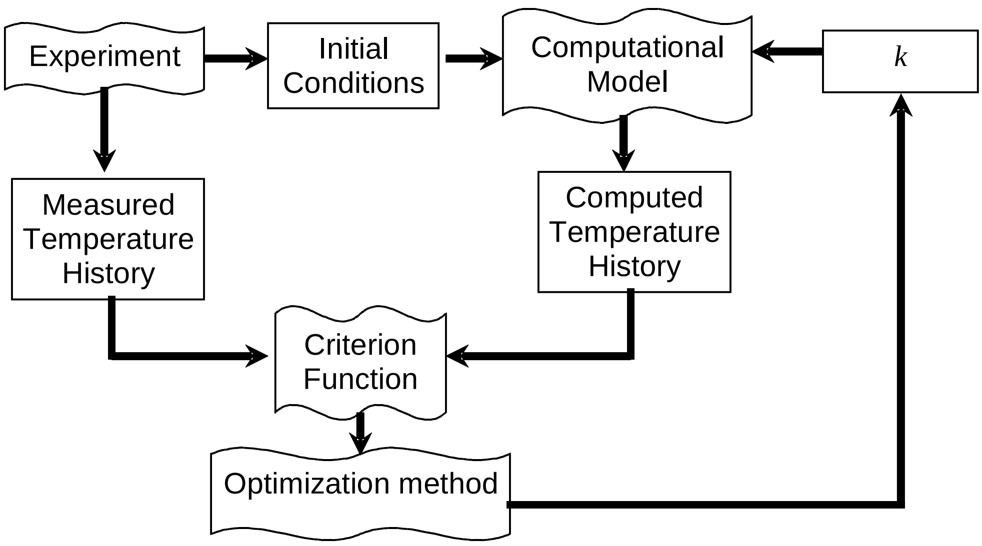

Figure 7 – Data flow during the identification process.

The identification method minimizes the error function in Eq. (2). First, an experiment is performed to obtain initial conditions and the measured temperature history inside the sensor. Using the initial conditions and the computational model, the temperature history is computed using HTC for a few time steps. This number of time steps is called number of forward time steps. Bigger distance of the installed thermocouple from the investigated surface requires bigger stabilization and thus larger number of forward time steps. The time dependent HTC on boundary can be described using a linear function during these several forward time steps. The computed and measured temperature histories are used in the criterion function Eq. (2). The minimum of this function is found using Brent's method (see Figure 8) or Downhill Simplex optimization method for multidimensional problems (see Figure 9). These optimization methods minimizes the Eq. (2) by changing the boundary conditions e.g. the *k* parameter that is direction of HTC (see Figure 7). The minimum of the criterion function represents the best HTC. For each step HTC and a new temperature history is computed.

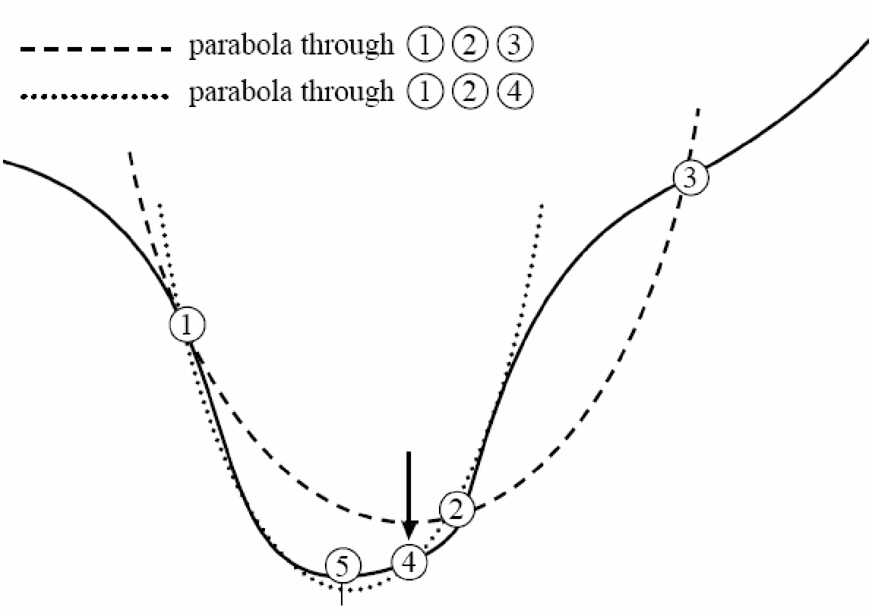

Figure 8 – Brent's method

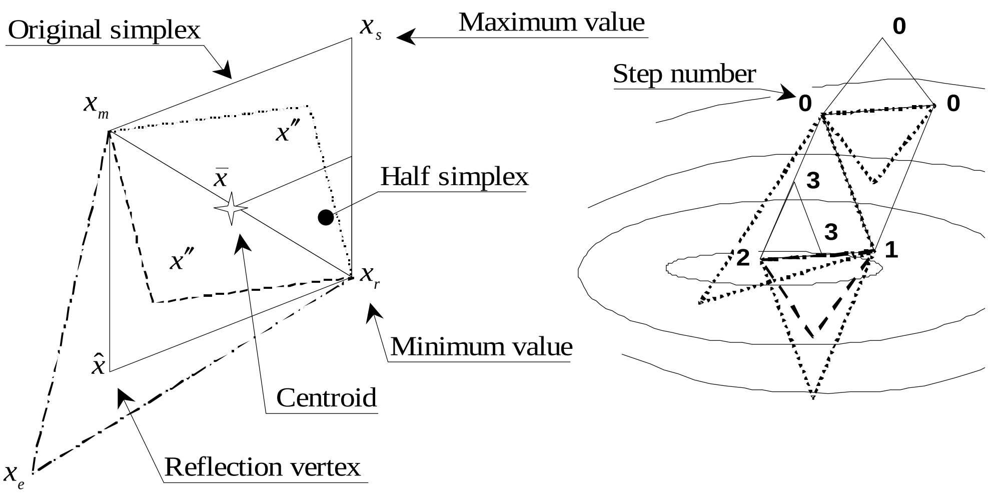

Figure 9 – Downhill simplex method

6. Results

Real measured data were taken to compare sequential Beck's approach with identification method. Tree different computations were made (see Figure 10). The first and the second computations used sequential Beck's approach with 5 and 15 forward steps, respectively. The third one used multidimensional optimization method. For the time when the HTC peak occurred the following equations were used as interpolating functions

$$HTC(x) = (\delta - \gamma) \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}} + \gamma$$
$$\sigma = \begin{Bmatrix} \sigma_L & \text{for } x < \mu \\ \sigma_R & \text{for } x \geq \mu \end{Bmatrix} \tag{11}$$

where the parameters δ and γ represent the maximum and minimum value of the HTC, respectively. The parameter σ describes the shape in the x direction. The shape for $x < \mu$, where μ represents the nozzle position, is different from $x \geq \mu$. Hence the parameter σ is divided into the two parameters σ_L and σ_R for the left and right sides, respectively.

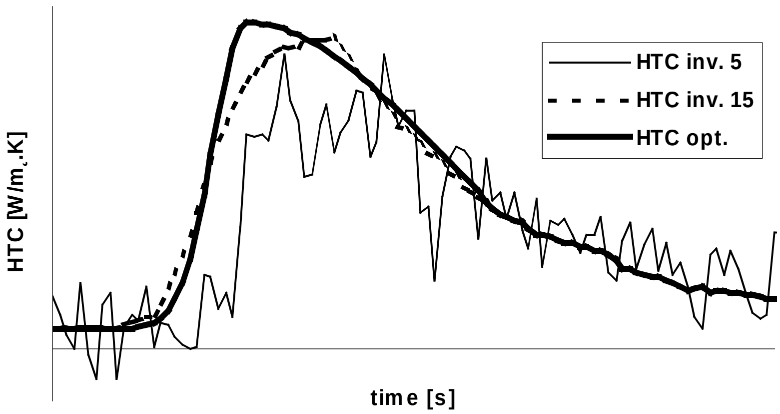

Figure 10 – Comparison of HTC history around HTC maximum.

Classical approach with 5 forward steps matched the measured temperature history almost perfectly. The RMS error was only 0.093 K and the maximum error was 0.25 K. But the computed HTC is very noisy because the HTC tries to follow all noise in the measured temperature history.

The computation with 15 forward steps shows that the noise can be quite well suppressed. But the computed temperature history does not follow the measured very well, the RMS error increased to 0.273 K and the maximum error was 0.96 K. High numbers of forward steps limit the maximum steep and also maximum value of the computed HTC. The shape of the HTC is also deformed and the maximum is moved to the right on time axis.

Identification method with interpolated curve removed noise in the computed HTC and matched very well the measured temperature history. The RMS error was 0.203 K, which is very close to the noise in measured data. The maximum error was only 0.38 K, which is much lower than in case of classical approach with 15 forward steps.

7. Conclusion

Mathematical procedures and precise inverse computations are used for evaluation of experimental results. Final output format of data are boundary conditions which can be used in numerical models of these processes. This technology makes it possible for engineers and scientists to construct more realistic mathematical models of physical processes.

The combination of two described methods allows increasing precision of inverse calculation. The methods allow evaluating long temperature records. The new investigative approach does not have the negative impact on stability of inverse task.

8. Acknowledgment

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9. Literature

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