

# METHOD OF ACCUMULATION CORE AND ITS USE BY MEASURING THERMAL PARAMETERS OF POROUS MATERIALS

Ivan Baník

*Physics Department of Faculty of Civil Engineering of Slovak University of Technology  
in Bratislava, 813 68 Bratislava, Radlinského 11  
e-mail: ivan.banik@stuba.sk*

## Abstract

The work describes and analyses a general view of an accumulation core method and deduces certain relations applicable for a thermal mode of permanent temperature increase. It discusses possible usage of the method by measuring thermo physical parameters of materials while presenting experimental results acquired with some heat-insulants.

## 1. INTRODUCTION

The work includes a proposal and theoretical analysis of a measuring method for thermo physical parameters of materials using so called accumulation core. The accumulation core is a body of very good thermal conductivity. Heat penetrates into the body from an outer metal block through a measured specimen (sample). Thermal differences in the volume of the metal block are to be ignored. The work deduces some general relations applicable for a method of accumulation core in case of permanent temperature increase. The method of accumulation core is an integral method. It is suitable for measuring parameters of heat insulants within a wide temperature range from low up to high temperatures.

## 2. ESSENTIALS OF THE METHOD

Essentials of the method of accumulation core are fully displayed in Fig 1. The measured specimen *S* is situated between two metal blocks – the outer metal block *MB* and the inner block - accumulation core *AC*. The inner block is labeled – accumulation core as it accumulates heat penetrating in through the sample from the outer metal block. Theoretically the blocks are of infinite thermal conductivity therefore the contact surfaces of the blocks with the specimen represent two isothermal surfaces of gradually changing temperature by time. Thus the accumulation core *AC* is a metal block fully immersed in the sample absorbing only the heat which penetrated the measured sample.

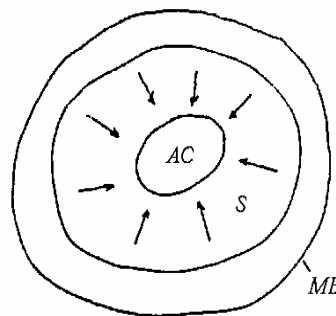


Fig. 1

Consequently the temperature of the accumulation core is slowly being changed. Based on the measured course of temperature of the outer metal block and the course of temperature of the accumulation core along with the known thermal capacity of the accumulation core and geometry of the specimen it is possible to determine the thermal conductivity of the specimen. The stated condition of negligibly small gradients of temperature in both of the metal blocks is easier to be met in actual practice in case of smaller temperature flows through the specimen and thus in case of specimen of low thermal conductivity. The method of accumulation core is a transient method. It is suitable for measuring parameters of heat insulants within a wide temperature range. Such measurements require knowledge of thermal dependence of thermal capacity of the accumulation core within a particular temperature range. In case of high thermal capacity of the accumulation core and thus adequately slow temperature increase the method could be considered quasi-steady.

### 3. TEMPERATURE CONDITIONS

Various temperature conditions could be applied by measuring with the accumulation core. At first, the general conditions will be discussed and afterwards particular conditions in more detail. The most general temperature conditions could use any temperature course  $T_2(t)$  of the outer block registered by a computer. Temperature course  $T_1(t)$  of the accumulation core is registered in the same manner as it represents response of the core to the outer block temperature changes. Following the changes of the border temperatures  $T_1(t)$  and  $T_2(t)$ , the known thermal capacity of the accumulation core and the known specimen geometry the thermal conductivity  $\lambda$  of the specimen can be specified. In practical realization of the idea the thermal conductivity of the specimen is considered to be a parameter searched by a computer to synchronize the theoretically defined temperature course of the accumulation core with the real measured temperature course. Differential method is used for the synchronization. A computer simulation of the temperature action in the system is used for the theoretical course of temperature of the accumulation core for a given value of the parameter  $\lambda$ .

Next, special temperature conditions of the steady temperature increase of the outer block are analyzed.

### 4. STEADY TEMPERATURE INCREASE CONDITIONS

Under these conditions the temperature of the outer block is changed linearly in time, in practice it increases linearly. Such increase of the border temperature all over the surface surrounding the specimen and the accumulation core steadily creates regular temperature conditions of the system characterized by linear temperature increase of all the specimen volume (at each point of the system) including the accumulation core. After having reached the regular state the created profile of the temperature field is "evenly" shifted towards higher temperatures while the speed of the temperature shift at each point of the system equals the speed of the temperature increase at the outer isotherm. Next, it will be shown that under the conditions of steady temperature increase a coefficient of the thermal conductivity  $\lambda$  of the specimen can be stated as

$$\lambda = \frac{Ac_j m_j \frac{dT'}{dt}}{\Delta T - \frac{B}{a} \frac{dT'}{dt}}$$

where the parameter  $c_j$  represents a specific thermal capacity of the accumulation core and  $m_j$  represents its weight. A composition of  $m_j c_j$  represents the thermal capacity of the accumulation core. The parameter  $a$  is a coefficient of the thermal conductivity of the specimen. The differential  $dT'$  means an elementary temperature increase at any point of the system and thus of the outer block temperature. The derivation  $dT'/dt$  indicates the speed of the outer block temperature increase. The constants  $A$  and  $B$  represent characteristic constants of a specific measuring arrangement independent from the speed of temperature increase, the thermal capacity of the core and the thermal parameters of the specimen. In case of analytical, or computer based numerical, determination of the value of the constants for a given system the upper mentioned relation enables determination of the coefficient of thermal conductivity  $\lambda$  for any speed of the temperature increase and for a specimen of different temperature parameters. It is sufficient to measure a difference  $\Delta T = T_2 - T_1$  between the temperature of the outer metal block (outer isotherm) and the temperature of the accumulation core (the inner isotherm) and determine the speed of the temperature increase. As the right side of the previous relation includes the coefficient  $a$  of the thermal conductivity of the specimen it seems impossible to determine the coefficient  $\lambda$  unless the first coefficient is known. However, a real situation in case of thermal – insulant porous materials is more favourable. When taking a closer look, it can be seen that the simplified relation

$$\lambda = \frac{A c_j m_j \frac{dT'}{dt}}{\Delta T}$$

which (unlike the previous relation) does not include a parameter of  $B/a$  term represents quasi-steady approximation with practically steady temperature field of the specimen. The simplified relation applies if the heat accepted by the specimen itself is insignificantly small comparing with the heat necessary to warm the more massive accumulation core and thus if the thermal capacity of the specimen is too small comparing with the thermal capacity of the core. The original relation is transformed to the simplified relation if the coefficient of the thermal conductivity and the specimen is of a very big value. From this point of view the presence of the parameter with the term  $B/a$  in the complete relation can be understood as a specification – correction of the quasi-steady method. In case of thermal insulants the mention correction tends to be several percent for the thermal capacity of the core. Importantly, a majority of a particular error could be eliminated by using an approximate value of the coefficient  $a$ . If the coefficient  $a$  is known with the accuracy of 20 percent a particular correction reduces the inaccuracy from its original e.g. five percent to one percent. A value of the coefficient  $a$  with a certain accuracy for a given material is usually known or can be defined. It is possible to define it based on data on measured thermal capacity, measured weight and  $\lambda$  value. A more accurate value of the coefficient  $a$  is to be determined upon more accurate  $\lambda$  value according to the upper mentioned complete relation. The specification can be done by progressive approaching.

## 5. DERIVATION OF THE THEORETICAL RELATION

The following formula applies for heat entering the system of “specimen + accumulation core” from the outer block in a time period  $dt$

$$dQ = \oint \vec{\varphi} \cdot d\vec{S} \cdot dt = -\lambda \cdot \oint \text{grad}T \cdot d\vec{S} \cdot dt$$

where  $\varphi$  is the density of the heat flow on the outer surface of the specimen and the surface “circle” diagram applies to the whole outer surface of the specimen. The stated heat is partly used to heat

the specimen itself and mostly to heat the accumulation core. The law of conservation of the energy for a regular mode with a steady temperature increase determines

$$dQ = c_j m_j dT' + cm dT'$$

The two previous formulas determine

$$\oint \text{grad} T \cdot d\vec{S} = -\frac{c_j m_j \cdot dT'}{\lambda \cdot dt} - \frac{V dT'}{adt}$$

where  $V$  is the volume of the specimen,  $a = \lambda/c\rho$ . The total density of the heat flow  $\varphi$  in the specimen can be divided in two components. The first component matches the heat proceeding to the accumulation core and the second component matches the heat proceeding to the specimen itself. Thus  $\varphi = {}^1\varphi + {}^2\varphi$ . Base on Fourier law we can write

$$\text{grad} T = (\text{grad} {}^1 T) + (\text{grad} {}^2 T)$$

where the first component represents the part of the field gradient responsible for the heat flow to the accumulation core (it matches the first term of the right side of the previous formula). Similarly, the second component  $\text{grad} {}^2 T$  is responsible for the heat flow to the specimen itself (and it matches the second term). Indeed, it is clear that just like the term  $\text{grad} T$  the temperature field itself can be divided. Thus the function

$$T(x, y, z, t) = {}^1 T(x, y, z, t) + {}^2 T(x, y, z, t)$$

because

$$(\text{grad} {}^1 T) + (\text{grad} {}^2 T) = \text{grad} ({}^1 T + {}^2 T)$$

The overall profile of the temperature field in the specimen depends also on the speed of temperature increase. In regular mode each of the gradient components is directly proportional to the speed of temperature increase. Thus we can say that the overall temperature difference  $\Delta T$  between the outer metal block and the accumulation core consists of two parts – a difference of the first component of the field and a difference of the second component of the field

$$\Delta T = \Delta {}^1 T + \Delta {}^2 T$$

The following formula applies for the heat entering the accumulation core in a unit of time

$$c_j m_j \frac{dT'}{dt} \approx \lambda \cdot (\Delta {}^1 T)$$

where  $\Delta {}^1 T$  is the temperature difference of the first component of the field. (By the given speed of temperature increase such difference would be formed in case that the thermal capacity of the specimen was zero.) A similar formula can be written for the second component of the temperature field related to the heat used to warm the specimen

$$cm \frac{dT'}{dt} \approx \lambda \cdot (\Delta {}^2 T)$$

The temperature difference  $\Delta {}^2 T$  would be formed between the blocks if the accumulation core was of zero thermal capacity but very big thermal conductivity and if the thermal capacity of the specimen was non-zero. The two last proportions can be defined by equations, but it is needed to establish relevant proportion constants. On the ground of some reasons the constants are different

for both of the formulas as the profile of the first and of the second temperature field are different. The proportions constants will be labeled  $A$  and  $B$ . The relevant formulas can be written

$$A.c_j m_j \frac{dT'}{dt} = \lambda.(\Delta^1 T)$$

$$B.cm \frac{dT'}{dt} = \lambda.(\Delta^2 T)$$

If we formulate the relevant temperature differences present on the right sides and consequently add these differences it results in the overall temperature difference  $\Delta T$  of the metal blocks

$$\Delta T = \Delta^1 T + \Delta^2 T = \frac{A}{\lambda} c_j m_j \frac{dT'}{dt} + \frac{B}{\lambda} c.m \frac{dT'}{dt}$$

As  $m = \rho V$ ,  $a = \lambda/c\rho$ , the last formula is

$$\Delta T = \frac{A}{\lambda} c_j m_j \frac{dT'}{dt} + \frac{B.V}{a} \frac{dT'}{dt}$$

It determines the relation for the coefficient of the thermal conductivity which we wanted to reason.

$$\lambda = \frac{A c_j m_j \frac{dT'}{dt}}{\Delta T - \frac{B}{a} \frac{dT'}{dt}}$$

If the thermal capacity of the accumulation core is very big and the temperature difference  $\Delta T$  between the outer block and the accumulation core is kept steady the speed of the temperature increase is very low. In such a case the second term in the denominator (containing the constant  $B$ ) is very small comparing with the first and it is to be considered insignificant. The last formula turns into a simplified form actually describing a quasi-steady process.

If the thermal capacity of the core is zero the numerator of the last formula is zero. As the coefficient  $\lambda$  is generally non-zero the denominator of the last fraction must be zero, which implies

$$a = \frac{BV}{\Delta T} \frac{dT'}{dt}$$

The relation applies only for a fictitious case of the accumulation core of zero thermal capacity. However, it can be used by real measuring to determine the constant  $B$  for a particular arrangement. Then we computer model a thermal action in a system with such a fictitious core (without a thermal capacity) by a given speed of temperature increase and known volume  $V$  of the specimen and selected coefficient  $a$ . Based on computer modeling we can determine the temperature difference  $\Delta T$  in regular mode and consequently determine the constant  $B$ .

$$B = \frac{a.\Delta T.dt}{V.dT'}$$

A special one is a case when the accumulation core actually does not exist – it shrinks to be a point.  $\Delta T$  determines the temperature difference between the block  $MB$  and a given point of the specimen. The whole cavity is filled with the specimen.

## 6. THE HEATED ACCUMULTION CORE

In case the accumulation core is heated for instance by an electric stove of thermal output  $P$  the formula for the coefficient of the thermal conductivity is

$$\lambda = \frac{A (c_j m_j \frac{dT'}{dt} - P)}{\Delta T - \frac{B}{a} \frac{dT'}{dt}}$$

It implies that if the temperature of the outer block is stabilized the stove of the output  $P$  heats the accumulation core to a stable temperature increased by  $\Delta T$  where (as  $dT'/dt = 0$ ) the parameter  $\lambda$  is

$$\lambda = \frac{-AP}{\Delta T} = \frac{AP}{T_1 - T_2}$$

This formula determines the constant  $A$ . Computer modeling determines the value of the stable difference  $\Delta T$  between the heated accumulation core (by selected output  $P$ , and selected conductivity  $\lambda$ ) and the outer block. The constant  $A$  is given by the relation  $A = \lambda (T_1 - T_2)/P$ . In case of easier geometry the constants  $A$  and  $B$  can be determined analytically.

## 7. THE PLATE ACCUMULTION CORE

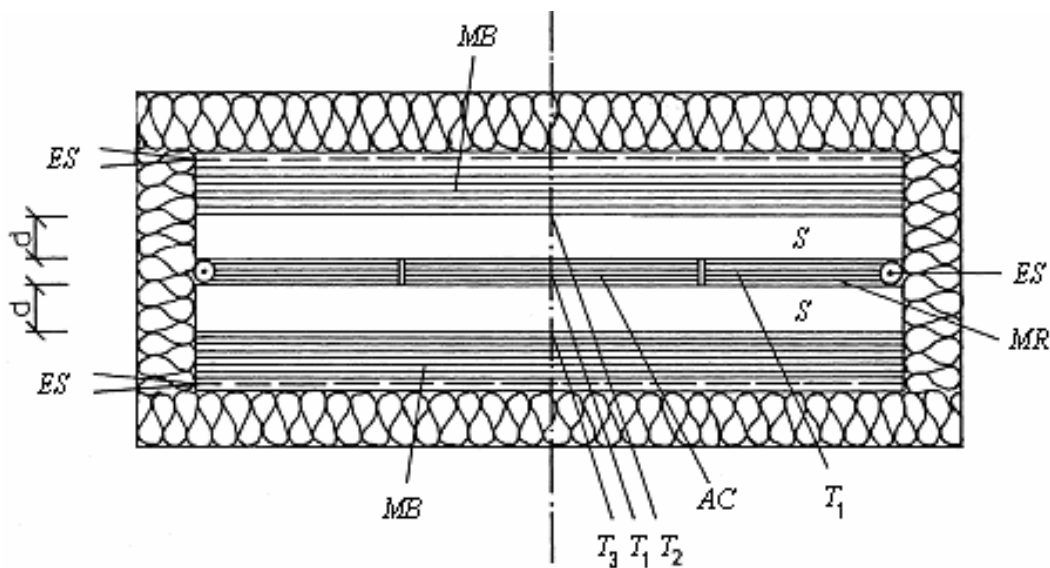


Fig. 2

Heretofore we have considered the accumulation core of the ordinary type all immersed in the specimen while the outer metal block created a cavity. However, the accumulation core  $AC$  could be of other shapes. It could have a shape suitable for a single-dimensional temperature flow. Such an accumulation core  $AC$  is for instance of the shape of a circular plate and is put between two plate specimen  $S$  with the outer surfaces heated by bigger metal blocks – metal plates  $MB$ . Heat enters the accumulation core – the circular plate vertically from both sides. The side temperature flow to the accumulation core must be eliminated by a cylinder shield – a metal ring with a regulated heat. The outer metal blocks – bigger metal plates are heated by computer controlled electric stoves  $ES$ . The course of temperatures in three points is registered.

## 8. EXPERIMENTAL RESULTS

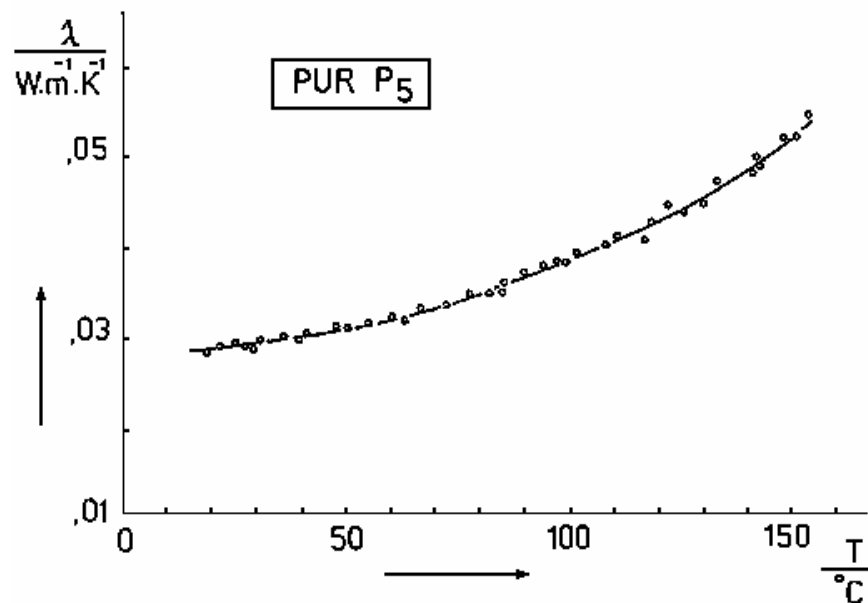


Fig. 3

The method of accumulation core has been used to measure thermal parameters of a thermo-insulator. On the fig. 3 we can see the dependence of the thermal conductivity coefficient  $\lambda$  of commercial polyurethane PUR 5 on temperature.

## REFERENCES

- [1] Luikov A.V.: Heat and Mass Transfer in capillary-Porous Bodies, Pergamon Press, Oxford, 1966
- [2] Krempaský J.: The measurements of the thermophysical Properties, SAV, Bratislava, 1969
- [3] Vasiliev L.L., Tanaeva S.A.: Thermal properties of porous materials, Minsk, Nauka I tehnika 1971 (in russian)
- [4] Mrlík F.: Moisture induced problem of building materials and construction (in slovak), Alfa, Bratislava, 1985
- [5] Jarny Y., Ozisik M.N., Bardon J.P.: A general method using adjoining equation for solving ultidimensional inverse heat conduction, Int. J. Heat Mass Transfer 34, 1991, p.2911
- [6] Alifanov O.M.: Solution of an inverse problem of heat conductivity by iteration method, J. Engineering Physics 26, 1972, p. 471