ANALYSIS OF ACCURACY OF THE MEASUREMENT OF THE SPECIFIC HEAT CAPACITY BY THE EDPS METHOD

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Abstract:
The method of the determination of uncertainty of the heat capacity measured by the EDPS method is presented in the paper. The apparatus for measurements was built in the Thermophysical laboratory at CPU in Nitra. The combined uncertainty of the heat capacity of the measurements was calculated and it is approximately equal to 4.5 %.

Keywords:
uncertainty, extended dynamic plane source (EDPS)

INTRODUCTION

The extended dynamic plane source method is a transition method which measures thermal conductivity and thermal diffusivity. This method is established for the analysis of the one-dimensional thermal flow in a final dimension sample with an ideal thermal plane source, [1].

The uncertainty of measurement is a parameter described by the range of values close to the results of the measurement which can be allocated to the set of values of the measured quantity. Uncertainty of the result of measurement consists of a few components which may be incorporated into the two categories:

A – the uncertainties evaluated by statistical methods,

B – the uncertainties evaluated by other methods.

The standard uncertainties of the type A (u_A) is obtained from the repeated measurements of value the same quantity. The standard uncertainties of the type B (u_B) is obtained by different methods. A characteristic property of the uncertainties of the type A is that the value of the uncertainty decreases with the higher number of repeated measurements. The value of the uncertainties of the type B does not depend on the number of repeated measurements.

The equivalence of the estimation of both types of uncertainities permits to merge all the standard uncertainties (types A and B) to a single value – the combined standard uncertainty. It is characterized by a numerical value obtained by the application of the method to connect the differences, and its components are expressions in the form of standard deviations.

Uncertainty as a measurable attribute is relatively new in the history of measurements, although errors and the analysis of errors has been the component part of the measuring accuracy techniques for a long time [2].

The aim of this paper is to introduce the way of determination of the uncertainties of measurement of the thermophysical properties of solids by the EDPS method.

Description of the Apparatus

A thermal plane source of a circular shape with the diameter $3 \cdot 10^{-2}$ m is placed in the plane $x = 0$ between two identical samples with the thickness $2L$, placed in positions $-L < x < L$. The samples have the shape of a disc and are of the same diameter as the thermal plane source. The samples placed between alumina blocks with the positions $x = -L$ and $x = L$ play the role of a heat conductive material (Fig.1). The thermal plane source plays the role of a thermal scanner and is made of nickel foil. Herewith the experimental arrangement is considerably simplified. The thermal plane source is covered with capton foil to prevent the mechanical damage.

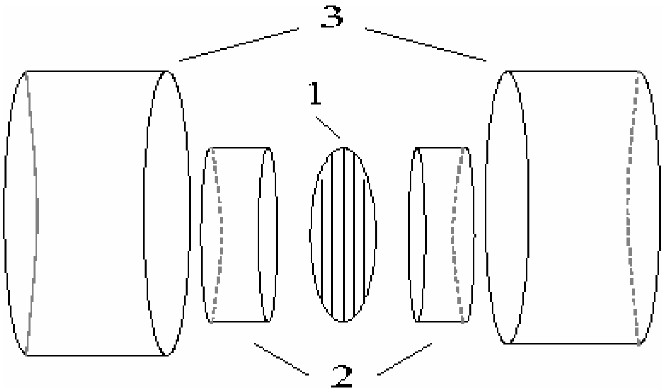

Fig. 1 - Experimental arrangement: (1 – thermal planer source, 2 – samples, 3 – blocks of alumina).

We assume the following ideal experimental conditions during the experiment:

- an ideal noninvertible thermal source – with a minor thickness and mass,
- an ideal contact of the samples with the source,
- zero thermal deficits from the edge of the samples,
- zero thermal resistance between the surface of the samples and the metal blocks [3].

If the tested solid is exposed to a constant thermal flow q, then the relation for the thermal function can be written as

$$T(t) = \frac{qL}{\lambda}\sqrt{\frac{t}{\pi\Theta}}\left(1 + 2\sqrt{\pi}\sum_{n=1}^{\infty}\beta^{n}\,\mathrm{ierfc}\left(n\sqrt{\frac{\Theta}{t}}\right)\right) \tag{1}$$

where q is the density of the thermal flow and λ is the thermal conductivity. Θ is the characteristic time of the sample given by relation

$$\Theta = \frac{L^2}{a}, \tag{2}$$

where L is the thickness of the sample and a the thermal diffusivity. The parameter β expresses the imperfection of the absorber of the heat and ierfc is the integral of the error function given by [4]

$$\mathrm{ierfc}(z) = \sqrt{\pi}\, e^{\left(-z^2\right)} - z\frac{2}{\sqrt{\pi}}\int_{z}^{\infty} e^{\left(-u^2\right)}\mathrm{d}u\,. \tag{3}$$

For the time $t < 0.3\Theta$ the relation (1) can be simplified and the thermal function will have the form

$$T(t) = \frac{q}{\lambda}\sqrt{\frac{at}{\pi}}, \tag{4}$$

which describes one-dimensional thermal flow in the infinite homogeneous surroundings.

The simplified form (4) permits to calculate the thermal effusivity of the sample from the guideline of the graphical relativity $T(t)$ by $\sqrt{t}$,

$$e = \frac{\lambda}{\sqrt{a}} = \sqrt{\rho c \lambda} \,. \tag{5}$$

After the simple mathematical adjustments we obtain the relation for the specific heat

$$c = \frac{\lambda}{a\,\rho}, \tag{6}$$

where ρ is the density of the tested solid.

During the experiment we must not forget the influence of the error caused by the breach of ideal conditions of the experiment. In our case they were: the minor thickness, the mass of the thermal plane source, and the ideal thermal contact with the sample. For the precise determination of the thermophysical parameters only a part of the measured thermal function assigned by range (Θ_D, 0.7Θ) is appropriate (here $\Theta_D \approx 5\,\mathrm{s}$ is the characteristical time of the disc and $t < 0.7\Theta$ is the time period in which nominal thermal loss from lateral surfaces of samples occurs). Exact results can be obtained from the relation (1) and the method of least squares, with the help of which we can determine the parameters λ and Θ[1].

DETERMINATION OF THE UNCERTAINITY OF THE MEASUREMENT OF THE SPECIFIC HEAT CAPACITY

We determine the uncertainty of the measurement of the specific heat capacity. A sample was made from acrylic plastic and had the spape of a disc. The weight of the sample, $m = 2446.406\,\mathrm{g}$ with the uncertainty $u(m) = 0.02\,\mathrm{g}$ was determined by the digital scales KERN EW 600 – 2M. The diameter and the length of the sample was measured by an adjustable meter with the resolution $0.01\,\mathrm{mm}$. As the sample does not have in the all places neither identical thickness nor length, we determined the uncertainty calculated by method of type A by repetition of the measurement of the thickness and length in the ten randomly selected places of the sample.

For the determination of the specific heat capacity we used the apparatus from Fig.1. The specific heat capacity measured by the EDPS method is given by the relation (6). With this apparatus we made ten measurement of the coefficients of the thermal conductivity and thermal diffusivity with the precision $5{\cdot}10^{-4}$ $\mathrm{W\,m^{-1}\,K^{-1}}$ and $5{\cdot}10^{-10}$ $\mathrm{m^2\,s^{-1}}$, respectively. In order to evaluate the average value of the specific heat capacity, we start from the relation

$$c = \frac{\lambda\,\pi\,d^2 h}{4\,a\,m} \tag{7}$$

obtained by the substitution of the relation for the density to the relation (7), where *h* is the thickness, *d* the diameter, and *m* the mass of the sample. The measured values of the thickness *h*, diameter *d*, coefficient of the thermal conductivity λ, and coefficient thermal diffusivity *a* of the sample are presented in Tab.1.

Tab.1 - Values of the thickness, diameter, coefficients of the thermal conductivity and thermal diffusivity of the sample.

h [mm]	d [mm]	λ [W m^{-1} K^{-1}]	a [$\cdot 10^{-7} m^2 s^{-1}$]
2.90	30.40	0.194	1.27
2.89	30.21	0.190	1.18
2.89	30.40	0.193	1.25
2.89	30.43	0.194	1.28
2.90	30.38	0.198	1.28
2.90	30.19	0.196	1.23
2.89	30.27	0.196	1.24
2.90	30.42	0.197	1.29
2.90	30.70	0.197	1.28
2.90	30.40	0.197	1.28

We determine the average values $\overline{h} = 2.896$ mm, $\overline{d} = 30.38$ mm, $\overline{\lambda} = 0.1952$ W m^{-1} K^{-1} , and $\overline{a} = 1.258 \cdot 10^{-7} m^2 s^{-1}$ from the relation

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \ , \tag{8}$$

using the values from Tab.1. The average value of the specific heat capacity as yielded by the relation (7) is $\overline{c} = 1372.86$ J kg^{-1}K^{-1} [5].

Since an indirect measurement of one quantity is involved, we determine the uncertainty of specific heat capacity measured by the EDPS method in the following way.

If $X_1, X_2, X_3, ..., X_p$ are directly measured quantities ($p < m$) and $X_{p+1}, X_{p+2}, ..., X_m$ are other input quantities whose estimations $x_{p+1}, x_{p+2}, ..., x_m$ are known from others sources (certificate, technical documentation, tablets, technical norm, calibration papers, documentation from producers, etc) and every directly measured quantity $X_1, X_2, X_3, ..., X_p$ is measured n – times, the standard uncertainty of the type A is determined from the relation

$$u_{Ay}^2 = \sum_{i=1}^{p} A_{x_i}^2 u_{Ax_i}^2 + 2 \sum_{i=1}^{p-1} \sum_{j>i}^{p} A_{x_i} A_{x_j} u_{Ax_{ij}} \ , \tag{9}$$

where A_{x_i} are the sensitivity coefficients satisfying

$$A_{x_i} = \left. \frac{\partial F(X_1, X_2, ..., X_m)}{\partial X_i} \right|_{X_1 = x_1, ..., X_m = x_m} \tag{10}$$

and

$$u_{Ax_i}^2 = \frac{1}{n(n-1)} \sum_{k=1}^{n} (x_{ik} - x_i)^2 \tag{11}$$

are the variances of the mean for the directly measured quantitied $X_1, X_2, X_3, ..., X_p$, where

$$x_i = \frac{1}{n}\sum_{k=1}^{n} x_{ik} \ , \tag{12}$$

and at the same time

$$u_{Ax_{ij}} = \frac{1}{n(n-1)}\sum_{k=1}^{n}(x_{ik} - x_i)(x_{jk} - x_j), \tag{13}$$

is the estimated covariance associated with the arithmetical averages x_i a x_j.

The standard uncertainty determined by method of the type B at the indirectly measured of the one quantity depend on standard uncertainties of direct measured quantities determined by method of the type B and on other standard uncertainties remaining input quantities determined by method of the type B. Then

$$u_{By}^2 = \sum_{i=1}^{p} A_{x_i}^2 u_{Bx_i}^2 + \sum_{i=p+1}^{m} A_{x_i}^2 u_{Bx_i}^2 + 2\sum_{i=1}^{p-1}\sum_{j>i}^{p} A_{x_i} A_{x_j} u_{Bx_{ij}} + 2\sum_{i=p+1}^{m-1}\sum_{j>i}^{m} A_{x_i} A_{x_j} u_{Bx_{ij}} \ , \tag{14}$$

where $u_{Bx_{ij}}$ is the covariance between the estimates x_i and x_j.

The covariance is determined from the relation

$$u_{x_{i,j}} = r_{x_{i,j}} u_{x_i} u_{x_j} \ . \tag{15}$$

The correlation coefficient r_x figure stage of the dependence between the estimates. The correlation coefficient may attain values from the interval $< -1,1 >$. The values approaching 0 correspond to a low dependence of the estimates, the values approaching 1 correspond to a high dependence of the estimates.

The combined uncertainty u_{Cy} is obtained by merging the uncertainty defined by the method of type A and by the method of type B with the help of the relation

$$u_{Cy} = \sqrt{u_{Ay}^2 + u_{By}^2} \ , \tag{16}$$

where the values of u^2_{Ay} and u_{By}^2 are obtained from the equations (9) and (14)[6, 7].

Tab.2 - The balance of uncertainties of the measurement of the specific heat capacity by EDPS method.

Source of uncertainties	Type	Stand. deviation $u(x_i)$	Sensitivity coeff. A_i	$u\ (c)$ JK^{-1}kg^{-1}
Meas. of the thickness	A	$5.1\cdot10^{-5}$ m	$46\cdot10^{4}$ Jm^{-1}K^{-1}kg^{-1}	23.46
	B	$6\cdot10^{-6}$ m	$46\cdot10^{4}$ Jm^{-1}K^{-1}kg^{-1}	2.76
Meas. of the diameter	A	$4.53\cdot10^{-4}$ m	$876\cdot10^{2}$ Jm^{-1}K^{-1}kg^{-1}	3.97
	B	$6\cdot10^{-6}$ m	$876\cdot10^{2}$ Jm^{-1}K^{-1}kg^{-1}	0.53
Meas. of the mass	B	$2\cdot10^{-5}$ kg	-411.74 Jm^{-1}K^{-1}kg^{-1}	$-83\cdot10^{-4}$
Meas. of the thermal cond.	A	$7.72\cdot10^{-4}$ Wm^{-1}K^{-1}	68240 m s kg^{-1}	52.64
	B	$3\cdot10^{-4}$ W m^{-1} K^{-1}	68240 m s kg^{-1}	20.47
Meas. of the thermal diff.	A	$1.07\cdot10^{-9}$ m^{2}s^{-1}	-106108 J K^{-1}kg^{-1} m^{-2}s	$-129.5\cdot10^{-5}$
	B	$3\cdot10^{-10}$ m^{2}s^{-1}	-106108 J K^{-1}kg^{-1} m^{-2}s	$-3.18\cdot10^{-5}$
Combined uncertainty				61

The combined uncertainty of the measurement of the specific heat capacity measured by the EDPS method $u(c)$ = 61 $JK^{-1}kg^{-1}$ was determined by means of the presented relations and the values given in Tab.2. The combined uncertainty introduces approximately 4.5 % of the average value of the specific heat capacity. In calculations we take into account the correlation between the estimations of the thickness and the diameter of the sample and the correlation between thermal conductivity and thermal diffusivity. As we used the same measuring instrument to measure the thickness and diameter as well as thermal conductivity and thermal diffusivity of the sample, the correlation coefficient $r(h, d) = 1$ a $r(\lambda, a) = 1$. Then for the specific heat capacity of the measured sample we obtain $u(c) = (1372.86 \pm 61)$ $JK^{-1}kg^{-1}$.

CONCLUSION

The result of this paper was the determination of the uncertainties of the measurement of the specific heat *capacity* by the EDPS method at the apparatus constructed in the Thermophysical laboratory at CPU in Nitra. A short description of the apparatus and technique for the determination of the uncertainty at the indirect measurement of one quantity was presented. We determined the combined standard uncertainty of the measurement of the specific heat capacity EDPS method, which is numerically equal to 4.5 % of its average value.

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