APPLICATION OF MODAL ANALYSIS TO IDENTIFICATION OF BUILDING THERMAL PARAMATERS

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Abstract:
An analytical method using the integral-transform technique is applied to solve the non-steady state heat transfer in buildings. The solution was used at the identification of building thermal parameters. The achieved results were compared with the results of an experimental identification.

1. INTRODUCTION

In the last thirty years a great number of calculation models for building thermal performance simulation have been created. The following general information on their accuracy and applicability are known:
- The objective of a designer is to search an optimum method of the analysis, dependent on the output requirements and the input data and parameters availability. Very detailed and universal methods require more parameters and inputs than simpler methods.
- The simplified models require less parameters and input data however they have a disadvantage as they are applicable only for such building types for which they were developed.
- The thermal performance simulation on a satisfactory level can be provided by the adequate heat performance model. Also in a case of the simplified system the most important characteristics of the heat transport system should be identified reliably.

The calculation method suitable for a design and assessment of buildings requires a simple heat performance model, concerning the whole buildings or thermal zones, without strong restrictions in a geometry, materials, structures and use. The variant energy consumption and indoor temperature courses are the required outputs of this model. As the building thermal performance is only one of the many aspects of the whole building design, the simple model with a reasonable calculation time is still preferred.

The most simple non-steady state building thermal performance model is the model with two nodal points, usually described by the 1st order ordinary differential equation with constant coefficients, using the hourly input data. The ratio of the parameters of this equation is a time constant of a system which is considered to be a measure of a building thermal inertia.

Usually the product of total thermal capacity and total thermal resistance of a building is considered as a measure of the building thermal inertia. The product is easily calculable and its significance as a thermal parameter of building dependent on its mass and heat loss is easily understood. In cases when almost a whole building thermal capacity is concentrated in indoor structures, whilst their thermal conductivity is higher than the conductivity of an envelope the product equals the thermal time constant of a building. In practice that means that the thermal performance of a building calculated as the solution of systems of the partial differential equations can be in certain cases replaced by the ordinary differential equation with easily calculable parameters.

The simplified dynamic model of the thermal performance of building will be verified by its development from the detailed mathematical model represented by the system of partial differential equations and energy balance equation for an indoor air. The simplified model is represented by the ordinary differential equation, developed from a detailed mathematical model using the integral-transform technique.

2. Equations of Non-Steady Heat Transfer Between Building Enclosure and Outdoor Environment

The description of a thermal performance of building will concern the mathematical model of a system represented by a building element (room) consisting of planar building structures, creating the indoor environment among which the radiation and conduction heat transfer acts. Thermophysical properties of the structures are considered to be constant in time and homogeneous in each layer. The air change rate in the enclosure is considered to be constant. The selected building elements are exposed to the excitations at the building surfaces (convection, radiation) and into the indoor air (heating input, auxiliary heat gains). In the solution of this system the following simplifying assumptions are used: linearity and stationarity of the system, homogeneity of structural layers, one-dimensional heat conduction through structures, a constant air temperature in the space and its instantaneous mixing, grey surfaces. The model does not consider a moisture transfer and phase changes.

The model consists of the system of partial differential equations of the heat conduction and the energy balance equation for an indoor air, together with the boundary equations complemented by the restrictions given to parameters and variables.

2. 1 Heat conduction equation

For each layer – l of each structure – k the partial differential heat conduction equation is valid:

$$\frac{\partial t_{k,l}(x,\tau)}{\partial \tau} = a_{k,l} \frac{\partial^2 t_{k,l}(x,\tau)}{\partial x^2} \tag{1}$$

in the region $0 < x_{k,l} < d_{k,l}$ for $\tau > 0$, where:

k = 1, 2, ... M
l = 1, 2, ... N

which is supposed without internal heat sources in a structure.

2. 2 Indoor air energy balance equation

The indoor air temperature, considering the introduced simplifying assumptions, is governed by the differential equation:

$$C_i \frac{dt_i(\tau)}{d\tau} + \Phi_i(\tau) = 0 \tag{2}$$

where: $\Phi_\iota(\tau)$ represents the total heat flow into the air
C_i is the total indoor air heat capacity

3. BOUNDARY CONDITIONS

The boundary conditions are defined for both surfaces of each wall and for the indoor air. Their number is then 2M + 1, where M is the number of walls. Here the interstitial boundary conditions are included.

3. 1 External walls surfaces

The heat flow penetrating through an external wall surface – k is the result of:

- convective heat transfer: α_{eck} $[t_e(\tau) - \tau_{k,1}(0,\tau)]$
- long wave radiation transfer between external surface and outdoor environment, which will, after the linearization be expressed by the relation α_{erk} $[t_{er}(\tau) - \tau_{\kappa,1}(0,\tau)]$
- direct and diffuse solar gains $E_{k,1}(0,\tau)$.

The total heat flow, penetrating through the external surface to the wall – k is then obtained from the expression:

$$\Phi_{k,1}(0,\tau) = [\alpha_{eck} \,.\, t_e(\tau) + \alpha_{erk} \,.\, t_{e,r}(\tau) + E_{k,1}(0,\tau)].S_k - (\alpha_{eck} + \alpha_{erk}) \,.\, S_k.t_{k,1}(0,\tau) \tag{3}$$

which can be written as:

$$\Phi_{k,1}(0,\tau) = -\,\alpha_{ek}.S_k \,.\, t_{k,1}(0,\tau) + \Phi'_{k,1}(0,\tau) \tag{4}$$

where α_{ek} is the surface convective and linearized long-wave radiation heat transfer coefficient
$\Phi'_{k,1}(0,\tau)$ represents the part of the heat flow $\Phi_{k,1}(0,\tau)$ independent on the building thermal state

The boundary condition at the external surface (x = 1, l = 1) has a general form:

$$S_k \lambda_{k,1} \frac{\partial t_{k,1}(0,\tau)}{\partial x} + \alpha_{ek} S_k t_{k,1}(0,\tau) = \Phi'_{k,1}(0,\tau) \tag{5}$$

for $\tau > 0$.

3. 2 Wall layers interstices

Under the assumption that thermal contacts between particular layers is ideal, the temperatures and heat flows continuity, penetrating the layers l and l + 1 of the wall – k is described by:

$$t_{k,l}(d,\tau) = t_{k,l+1}(0,\tau) \tag{6}$$

$$\lambda_{k,l} \frac{\partial t_{k,l}(d,\tau)}{\partial x} = \lambda_{k,l+1} \frac{\partial t_{k,l+1}(0,\tau)}{\partial x} \tag{7}$$

3. 3. Internal wall surfaces

Solving the complex heat transfer problem in the interior the assumption of linearity is suitable. Furthermore the components of heat flows impinging the internal surfaces dependent on the thermal state of room will be separated from the components independent on the thermal state, computable as a part of external excitations. The former components represent thermal bonds among particular structures. The total thermal flow impinging to the internal surface includes

various excitations (solar gains, part of heating input, auxiliary heat gains) and also the heat transfer by:
- the infrared radiation among internal surfaces
- the natural convection among indoor air and internal surface of the room.

3. 4 Vector of long-wave radiation heat flows among internal surfaces of the room structures

The infrared heat transfer among M grey internal surfaces is given by the equation:

$$\mathbf{E} \cdot \mathbf{J} = \mathbf{e} \tag{8}$$

$$\mathbf{F} \cdot \mathbf{J} = \mathbf{\Phi}_i^r \tag{9}$$

where: $\mathbf{\Phi}'_r$ is the vector of a total heat flux through surfaces 1 – M
$\mathbf{J}$ is the vector of heat flux densities by long wave radiation among surfaces 1 – M

$$\mathbf{F} = \begin{Vmatrix} \sum_{k=1}^{M} S_1\varphi_{1k} & -S_1\varphi_{12} & \cdots & -S_1\varphi_{1M} \\ -S_2\varphi_{21} & \sum_{k=1}^{M} S_2\varphi_{2k} & \cdots & -S_2\varphi_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ -S_M\varphi_{M1} & -S_M\varphi_{M2} & \cdots & \sum_{k=1}^{M} S_M\varphi_{Mk} \end{Vmatrix} \tag{10}$$

S_k is the surface area of structure - k
φ_{kk} is the shape factor for radiation from surface k to surface - k

$$\mathbf{E} = \mathbf{F} + diag\left[S_1 \frac{\varepsilon_1}{1-\varepsilon_1}, S_2 \frac{\varepsilon_2}{1-\varepsilon_2}, \cdots, S_M \frac{\varepsilon_M}{1-\varepsilon_M} \right] \tag{11}$$

$$\mathbf{e} = [e_1 T_{1,N}(d,\tau),\ e_2 T_{2,N}(d,\tau),\ \ldots,\ e_M T_{1,M}(d,\tau)]^T \tag{12}$$

$$e_k = S_k \frac{\varepsilon_k}{1-\varepsilon_k} \sigma \left[T_{k,N}(d,\tau) \right]^3 \tag{13}$$

ε_k is the emissivity of a surface – k
is the Stefan-Boltzmann constant
$T_{k,N}(d,\tau)$ is the absolute temperature of a surface – k

Under the assumption that it is possible to find the corresponding mean value e_k for each surface it is possible to express from equations (8) and (9):

$$\mathbf{\Phi}^r_p = \mathbf{F}.\boldsymbol{\varepsilon}^{-1}.\mathbf{e} = \mathbf{G}^r\, [T_{1,N}(d,\tau),\ T_{2,N}(d,\tau),\ \ldots,\ T_{N,N}(d,\tau)]^T \tag{14}$$

where:

$\mathbf{G}^r = \mathbf{F}.\boldsymbol{\varepsilon}^{-1} diag\, [e_1, e_2, \ldots, e_M]$ is the matrix of a long wave radiation heat transfer among the internal surfaces of a room.

The long wave radiation heat flows vector for internal surfaces of particular structures is then:

$$\mathbf{\Phi}^r_p(\tau) = \mathbf{G}^r.\mathbf{t}_p(\tau) - \mathbf{\Phi}^{r/}_p(\tau) \tag{15}$$

$$\mathbf{t}_p(\tau) = [t_{1,N}(d,\tau), t_{2,N}(d,\tau), ..., t_{M,N}(d,\tau)]^T \qquad (16)$$

is the vector of room internal surfaces temperatures, and:

$$\mathbf{\Phi}_p^{r/}(\tau) = \left[\Phi_{1,N}^{r/}(d,\tau), \Phi_{2,N}^{r/}(d,\tau), \ldots \Phi_{M,N}^{r/}(d,\tau)\right]^T \qquad (17)$$

where: $\Phi_{k,N}^{r/}(d,\tau)$ represents the part of radiation heat flow impinging to the surface – k, independent on the thermal state of a room.

3. 5 Vector of convective heat flows between indoor air and walls

The vector of convection heat flows between indoor air and internal room surfaces can be written strightly:

$$\mathbf{\Phi}^c{}_p(\tau) = \mathbf{G}^c.\mathbf{t_p}(\tau) - \mathbf{g}^c.\mathbf{t}_i(\tau) \qquad (18)$$

where:

$$\mathbf{G}^c = \text{diag}\,[S_1\alpha_{ic1}, S_2\alpha_{ic2, ...,} S_M\alpha_{icM}] \qquad (19)$$

$$\mathbf{g}^c = [S_1\alpha_{ic1}, S_2\alpha_{ic2, ...,} S_M\alpha_{icM}]^T \qquad (20)$$

α_{ick} is the internal convective surface heat transfer coefficient, supposed to be constant in time

$t_i(\tau)$ is the indoor air temperature supposed to be constant in space

3. 6 Heat flow into the room indoor air

The total heat flow penetrating into the indoor air includes:

- the internal gains $\mathbf{\Phi}^+{}_i(\tau)$
- the air change at the temperature t_e: $\mathbf{\Phi}_m.c_e[t_e(\tau) - t_i(\tau)]$
- the convective surface heat transfer for structure – k:

$$\alpha_{ick}.S_k\,[t_{k,N}(d,\tau) - t_i(\tau)] \qquad (21)$$

In general, then it can be written:

$$\mathbf{\Phi}_i(\tau) = \left(\sum_{k=1}^{M} \alpha_{ick} S_k + \Phi_m c_e\right) \cdot t_i(\tau) - \mathbf{g}^{cT}\mathbf{t}_p(\tau) - \Phi_{\mathbf{i}}^{/}(\tau) \qquad (22)$$

where:

$$\Phi^{/}{}_i(\tau) = \Phi^+{}_i(\tau) + \Phi_m.c_e.t_e(\tau) \qquad (23)$$

is the component independent on the thermal state of a room.

3. 7 General expression for indoor boundary conditions

The heat transfer in an interior can be on the basis of the parts 3.5 and 3.6 described by the matrix expression:

$$\mathbf{\Phi}(\tau) = \mathbf{G.t}(\tau) - \mathbf{\Phi}^{/}(\tau) \qquad (24)$$

which can be itemized with use of block matrixes in the form:

$$\left\| \frac{\mathbf{\Phi}_i(\tau)}{\mathbf{\Phi}_p^r(\tau)+\mathbf{\Phi}_p^c} \right\| = \left\| \begin{array}{c|c} \sum_{k=1}^{M} \alpha_{ick} S_k + \Phi_m c_e & -\mathbf{g}^{cT} \\ \hline -\mathbf{g}^c & \mathbf{G}^r + \mathbf{G}^c \end{array} \right\| \cdot \left\| \frac{\mathbf{t}_i(\tau)}{\mathbf{t}_p(\tau)} \right\| - \left\| \frac{\mathbf{\Phi}_i^{/}(\tau)}{\mathbf{\Phi}_p^{r/}(\tau)} \right\| \tag{25}$$

$\Phi(\tau)$, $\mathbf{t}(\tau)$,$\mathbf{\Phi}^{/}(\tau)$ are the vectors with M+1 elements.

The matrix g(M + 1, M + 1) is symmetrical with positive diagonal elements. Other its elements are negative. G is the matrix of thermal bonds among particular elements in the interior.

The boundary condition for internal surface of the wall k (l = N, x = d) in a general form is:

$$S_1\lambda_{1,N}\frac{\partial t_{1,N}(d,\tau)}{\partial x} + \alpha_{i1}S_1 t_{1,N}(d,\tau) = \Phi_{k,N}^{r/}(d,\tau) + \alpha_{ic1}S_1 t_i(\tau) + \sum_{k=2}^{M} G_{1k}^r \cdot t_{k,N}(d,\tau) \tag{26}$$

4. Solution with Use of Integral Transform Technique

For the solution of non-homogeneous heat conduction equations with non-homogeneous boundary conditions the method of integral transform is suitable. At the solution of a heat condition problem by this method with use of integral transform the second partial derivations according to the space coordinates are excluded and the heat conduction problem is reduced to the ordinary differential equation. The resulting differential equation is then solved for a transformed boundary condition of given problem. From the integral transform of temperature function obtained by this way it is possible easily to get the required solution of the problem. The method is developed from a classical method of the separation of variables.

4. 1 Separation of variables

Let us consider the solution of the heat transfer problem formulated by the equations (1) and (2) for a homogeneous version of boundary conditions (4) and (26). The solution of this problem supposes the separation of variables in the form:

$$t_{k,l}(x,\tau) = \Psi_{k,l}(\beta_n,x).\vartheta_{k,l}(\tau) \tag{27}$$

After putting the expression (27) into the equations (1) and (2) two separated differential equations result:

For the variable $\vartheta_{k,l}$:

$$\frac{d\vartheta_{k,l}(\tau)}{d\tau} + \beta_n \cdot \vartheta_{k,l}(\tau) = 0 \tag{28}$$

For the function $\Psi_{k,l}(\beta_n,x)$:

- for each layer – l of each wall - k

$$\frac{d^2\Psi_{k,1}(\beta_n,x)}{dx^2} + \frac{\beta_n\Psi_{k,1}(\beta_n,x)}{a_{k,1}} = 0 \tag{29}$$

- for the indoor air

$$\Phi_i(\tau) - \beta_n.C_i.\Psi_i(\beta_n).\ ).\vartheta_{k,l}(\tau) = 0 \qquad (30)$$

The index n expresses the fact that an infinite number of discrete values of β_1, β_2, β_3 … β_n and the corresponding functions exists.

The boundary conditions for equations (29) and (30) are obtained introducing the relation (27) into the homogeneous version of boundary conditions (4) and (26). Then it is valid:

- for the external surface of wall k

$$\Phi_{k,1}(0,\tau) + \alpha_{ek}S_k\Psi_{k,1}(\beta_n,0) = 0 \qquad (31)$$

- for the interstice of layers l and l + 1 of wall - k

$$\Psi_{k,1}(\beta_n,d) = \Psi_{k,l+1}(\beta_n,0) \qquad (32)$$

$$\lambda_{k,l}\frac{d\Psi_{k,1}(\beta_n,d)}{dx} = \lambda_{k,l+1}\frac{d\Psi_{k,l+1}(\beta_n,0)}{dx} \qquad (33)$$

- for the heat transfer in indoor space

$$\mathbf{\Phi = G\ .\ \Psi} \qquad (34)$$

The solution of the equations (28) and (29) for the boundary conditions (31) and (34) represents the problem of finding eigenvalues and the corresponding eigenfuctions. For the found discrete spectrum of eigenvalues and the corresponding eigenfunctions the representation of a temperature function by the eigenfunctions for the layer - l of the structure – k is considered in the form:

$$t_{k,l}(x,\tau) = \sum_{n=1}^{\infty} C_n(\tau)\cdot\Psi_{k,l}(\beta_n,x) \qquad (35)$$

when the summation is given for the whole discrete spectrum of eigenvalues, whilst:

$$C_n(\tau) = \frac{1}{N(\beta_n)}\sum_{k=1}^{M} S_k \sum_{l=1}^{N}\rho_{k,l}c_{k,l}\int_0^{d_{k,l}}\Psi_{k,l}(\beta_n,x)\cdot t_{k,l}(x,\tau)dx_{k,l} \qquad (36)$$

The relation (36) gives after the putting into (35) the expression for inverse transform:

$$t_{k,l}(x,\tau) = \sum_{n=1}^{\infty}\frac{\Psi_{k,l}(\beta_n,x)}{N(\beta_n)}\bar{t}(\beta_n,\tau) \qquad (37)$$

whilst the expression for integral transform is:

$$\bar{t}(\beta_n,\tau) = \sum_{k=1}^{M} S_k \sum_{l=1}^{N}\rho_{k,l}c_{k,l}\int_0^{d_{k,l}}\Psi_{k,l}(\beta_n x)\cdot t_{k,l}(x,\tau)dx_{k,l} \qquad (38)$$

$N(\beta_n)$ is the normalization integral:

$$N(\beta_n) = \sum_{k=1}^{M} S_k \sum_{l=1}^{N}\rho_{k,l}c_{k,l}\int_0^{d_{k,l}}\left[\Psi_{k,l}(\beta_n x)\right]^2 dx_{k,l} \qquad (39)$$

4. 2 Eigenvalue problem

The integration of the relation (29) for a wall enables to express the eigenfunction and the connecting heat flow at an internal surface of arbitrary wall as the function of heat flow and eigenfunction (equal to zero in this case) at the external wall surface:

$$\begin{Vmatrix} \Psi_{k,N}(\beta_n, d) \\ \Phi_{k,N}(\beta_n, d) \end{Vmatrix} = \begin{Vmatrix} I_k(\beta_n) & J_k(\beta_n) \\ K_k(\beta_n) & L_k(\beta_n) \end{Vmatrix} \cdot \begin{Vmatrix} 0 \\ \Phi_{k,1}(\beta_n, 0) \end{Vmatrix} \tag{40}$$

The matrix $\mathbf{M}_k$ with the elements I_k, J_k, K_k, L_k is the parametric function of β_n. It is the product of transfer matrices of each layer with the transfer matrix $\mathbf{M}_{k,e}$:

$$\mathbf{M}_k = \mathbf{M}_{k,N}.\ \mathbf{M}_{k,N-1} \ldots.\ \mathbf{M}_{k,1}.\ \mathbf{M}_{k,e} \tag{41}$$

where:

$$\mathbf{M}_{k,e} = \begin{Vmatrix} 1 & -\dfrac{1}{\alpha_{e,k} S_k} \\ 0 & 1 \end{Vmatrix} \tag{42}$$

The relation (40) results from the conditions of continuity of temperatures and heat flows between particular layers of a structure (31) - (33).
The transfer matrix of the layer – l and structure – k has the form:

$$\mathbf{M}_{k,1} = \begin{Vmatrix} \cos\sqrt{\dfrac{\beta_n}{a_{k,1}} d_{k,1}} & -\dfrac{1}{S_k \cdot \sqrt{b_{k,1}\beta_n}} \sin\sqrt{\dfrac{\beta_n}{a_{k,1}} d_{k,1}} \\ S_k \cdot \sqrt{b_{k,1}\beta_n} \cdot \sin\sqrt{\dfrac{\beta_n}{a_{k,1}} d_{k,1}} & \cos\sqrt{\dfrac{\beta_n}{a_{k,1}} d_{k,1}} \end{Vmatrix} \tag{43}$$

where: $b_{k,1}$ is the thermal admittance of layer l

From the expression (40) M pairs of the relations conclude:

$$\Psi_{k,N}(\beta_n,d) = J_k(\beta_n).\Phi_{k,1}(\beta_n,0) \tag{44}$$

$$\Phi_{k,N}(\beta_n,d) = L_k(\beta_n).\Phi_{k,1}(\beta_n,0) \tag{45}$$

For the indoor air of a room it is possible to write:

$$J_i(\beta_n) = \frac{1}{\beta_n C_i},\quad L_i(\beta_n) = 1 \tag{46}$$

Analogously:

$$\Psi_i(\beta_n) = J_i(\beta_n).\Phi_i(\beta_n) \tag{47}$$

$$\Phi_i(\beta_n) = L_i(\beta_n).\Phi_i(\beta_n) \tag{48}$$

Together with the relation (32) a set of the equations (44) – (48) can be written in the form of matrix equations:

$$\mathbf{\Psi} = \mathbf{J}(\beta_n).\mathbf{\Phi}_e \tag{49}$$

$$\mathbf{\Phi} = \mathbf{L}(\beta_n).\mathbf{\Phi}_e \tag{50}$$

$$\mathbf{\Phi} = \mathbf{G}.\mathbf{\Psi} \tag{51}$$

$\mathbf{\Phi}_e$, $\mathbf{\Phi}$, $\mathbf{\Psi}$ are vectors with M + 1 elements,
$\mathbf{J}(\beta_n)$ and $\mathbf{L}(\beta_n)$ are diagonal matrices.

From the above mentioned relations the following relationship results:

$$[\mathbf{G}.\mathbf{J}(\beta_n) - \mathbf{L}(\beta_n)]\mathbf{\Phi}_e = 0 \tag{52}$$

The condition of a non-zero solution of this relationship is the validity:

$$\det[\mathbf{G}.\mathbf{J}(\beta_n) - \mathbf{L}(\beta_n)] = 0 \tag{53}$$

The relation (53) represents the transcendent equation with the infinite number of real roots, which are equal to the eigenvalues of the problem defined by equations (29) and (33). For each known eigenvalue the linear system of homogeneous equations (52), solution of which is the vector β_n necessary for the calculation of eigenfunctions can be established.

4. 3 Integral transform of heat conduction problem

Determining the integral transform and the inverse integral transform a further step of the analysis is the exclusion of partial derivations according the space variables from the partial differential equation with use of integral transform (38). The system of ordinary differential equations for the transformed temperature will then result:

$$\frac{d\bar{t}(\beta_n,\tau)}{d\tau} + \beta_n \cdot \bar{t}(\beta_n,\tau) = \sum_{k=1}^{M+1} \Psi_{k,N}(\beta_n,d)\cdot \Phi'_{k,N}(d,\tau) + \sum_{k=1}^{M} \Psi_{k,l}(\beta_n,0)\cdot \Phi'_{k,l}(0,\tau) \tag{54}$$

The initial conditions for this equation can be obtained by the integral transform of the initial conditions.
The solution of the equation (54) by the variation of constants method for the initial condition $\bar{t}(\beta_n,0)$ gives the transformed temperature:

$$\bar{t}(\beta_n,\tau) = e^{-\beta_n \tau} \cdot \left(\bar{t}(\beta_n,0) + \int_0^\tau e^{\beta_n,\tau} \cdot \sum_{k=1}^{M}\sum_{l=1}^{N} \Psi_{k,1}(\beta_n,x)\cdot \Phi'_{k,l}(x,\tau) d\tau \right) \tag{55}$$

The inverse transform of the expression (55) gives the temperature distribution in structures of a room:

$$\begin{aligned} t_{k,l}(x,\tau) = \sum_{n=1}^{\infty} \frac{e^{-\beta_n \tau}}{N(\beta_n)} \cdot \Psi_{k,l}(\beta_n,x) \cdot \\ \cdot \left\{ \bar{t}(\beta_n,0) + \int_0^\tau e^{\beta_n,\tau} \cdot \left[\sum_{k=1}^{M} \Psi_{k,1}(\beta_n,0)\cdot \Phi'_{k,1}(0,\tau) + \sum_{k=1}^{M+1} \Psi_{k,N}(\beta_n,d)\cdot \Phi'_{k,N}(d,\tau) \right] d\tau \right\} \end{aligned} \tag{56}$$

The solution is valid for the boundary conditions of 3^{rd} kind at all surfaces. An alternative form of the solution (56) can be obtained by its integration by parts. Then the solution is decomposed into three groups of simpler solutions – one quasi-stationary and two non-stationary ones.

4.4 Heat transfer in room

The thermal performance of a room can be then described by two matrix equations issuing from the relations (54) and (56):

$$\frac{d\overline{\mathbf{T}}}{d\tau} + \mathbf{A} \cdot \overline{\mathbf{T}} = \mathbf{B} \cdot \mathbf{\Phi}^{/} \tag{57}$$

$$\mathbf{T} = \mathbf{C} \cdot \overline{\mathbf{T}} + \mathbf{D} \cdot \mathbf{\Phi}^{/} \tag{58}$$

$\overline{\mathbf{T}}$ and **T**	are the vectors of transformed temperatures and temperatures of the dimension ∞
B	is the matrix of the (∞, 2M + 1) type
A	is the quadratic diagonal matrix of dimension ∞, with elements $A_{nn} = (\beta_n)$
$\mathbf{\Phi}^{/}$	is the vector of 2M + 1 thermal excitations
C	is the matrix of type (M, ∞)
D	is matrix of type (M, 2M + 1)

The matrix **D** concerns the quasi-stationary component of the solution and it is given by the relation:

$$\mathbf{D} = \mathbf{C} \cdot \mathbf{A}^{-1}\mathbf{B} \tag{59}$$

The foursome of the matrices **A**, **B**, **C**, **D** is a complex characteristics of the heat transfer between the room and outdoor environment. The solution of the differential equation (57) can be obtained by the variation of constants method, if the thermal excitations have a simple course (the shape of step, triangle or sine).

5. Identification of Building Equivalent Thermal Parameters

If only the first eigenvalue is significant, the heat transfer in a room can be modelled by 1st order differential equation and indoor space energy balance equation with the following parameters: the total thermal resistance between indoor and outdoor temperatures - R_o, the total thermal resistance between the nodal point for indoor temperature and nodal point for internal structures - R_i and the total heat capacity of a room - C:

$$\frac{dt}{d\tau} + \frac{1}{R_i \cdot C} \cdot t = \frac{1}{Ri \cdot C} \cdot t_i \tag{60}$$

$$\frac{1}{R_i}(t_i - t) + \frac{1}{R_o}(t_i - t_e) - \Phi' = 0 \tag{61}$$

As the equations (60) and (61) are analogous to the equations (57) and (58), the following relationships among the parameters of detailed analytical and simplified equations are valid:

$$\mathrm{C}_1 \cdot \mathrm{A}_1^{-1} \cdot \mathrm{B}_1 = R_o \tag{62}$$

$$\mathrm{C}_1 \cdot \bar{t}(\beta_1, 0) = \frac{R_o}{R_i + R_o} \tag{63}$$

$$\beta_1 = \frac{1}{(R_o + R_i) \cdot C} \tag{64}$$

With use of the relationships (63) and (64) the parameters R_o, R_i and C were identified from a detailed heat transfer model and compared with the equivalent thermal parameters of various buildings obtained from the actual measured data. The supposed air change rate for all analysed objects was 1 h^{-1}.

5. 1 Object No. 1 movable residential cell

First eigenvalue: $\beta_1 = 5.10^{-5}\ s^{-1}$

Integral transform of the initial temperature: $\bar{t}(\beta_1,0) = 16.562 \cdot 10^6$

Normalization integral: $N(\beta_1) = 1.017.10^8$

Structure	1	2	3	4	5	6	7
$\psi_{k,N}(\beta_1,d)$	8.37	7.55	4.21	3.94	4.59	7.49	4.59
$\psi_{k,1}(\beta_1,0)$	0.54	0.55	0.90	1.12	1	5.29	1

For the indoor air $\psi_i(\beta_1) = 5.67$

Total heat loss at the unit temperature difference $1/R_o = 67$ W/K.

The contribution of the term corresponding to the first eigenvalue to the exact solution of indoor air temperature at the steady state and the constant unit diffuse solar radiation flow (0.0139 K) represents:

$$\frac{\Psi_i(\beta_1)\sum'_{\substack{k=1\\k\neq 3,4}}^{7}\Psi_{k,N}(\beta_1,d)\cdot\varphi_{4,k}}{\beta_1\cdot N(\beta_1)} = 0.0072K$$

which represents 52% of the exact value. That means that the thermal performance of the object can not be modelled satisfactorily by the expected ordinary differential equation. This is logical, as the building heat capacity is located in its envelope - not concentrated in indoor structures (as it was supposed in the introduction).

The contribution of the term corresponding to the first eigenvalue to the exact solution of indoor air temperature suddenly after the time $\tau = 0$ during the cooling with unit initial temperature is:

$$\frac{\Psi_i(\beta_1)\bar{t}(\beta_1,0)}{N(\beta_1)} = 0.92K$$

Equivalent thermal parameter	Analysis	Experiment
$1/R_o$ [W/K]	67	66
$1/R_i$ [W/K]	288	318
C [J/K]	1340000	1043288

5. 2 Object No. 2 classroom

First eigenvalue: $\beta_1 = 2.95^{-6}\ s^{-1}$

Integral transform of the initial temperature: $\bar{t}(\beta_1, 0) = 5.132 \cdot 10^8$
Normalization integral: N(β$_1$) = 49.318.10^6

Structure	1	2	3	4	5	6	7
$\psi_{k,N}(\beta_1,d)$	1.14	1.05	0.64	0.52	0.98	0.98	0.98
$\psi_{k,1}(\beta_1,0)$	1.18	1.06	0.04	0.14	1	1	1

For the indoor air $\psi_i(\beta_1)$ = 0.84
Total heat loss at the unit temperature difference 1/R$_o$ = 170 W/K

The contribution of the term corresponding the first eigenvalue to the exact solution of indoor air temperature at the steady state and the constant unit diffuse solar radiation flow (0,0059 K) represents:

$$\frac{\Psi_i(\beta_1)\sum_{\substack{k=1 \\ k\neq 3,4}}^{7} \Psi_{k,N}(\beta_1, d) \cdot \varphi_{4,k}}{\beta_1 \cdot N(\beta_1)} = 0.0061K$$

This is ca 100% of the exact value and confirms the neglecting all members of a series in the solution, instead of the first one.

The contribution of the term corresponding to the first eigenvalue to the exact solution of indoor air temperature suddenly after the time τ = 0 during the cooling with unit initial temperature:

$$\frac{\Psi_i(\beta_1)\bar{t}(\beta_1, 0)}{N(\beta_1)} = 0.91K$$

Equivalent thermal parameter	Analysis	Experiment
1/R$_o$ [W/K]	170	209
1/R$_i$ [W/K]	1719	1900
C [J/K]	57646660	59016384

5. 3 Object No. 3 bedroom

First eigenvalue β$_1$ = 2.38^{-6} s^{-1}
Integral transform of the initial temperature $\bar{t}(\beta_1, 0) = 5.132 \cdot 10^8$
Normalization integral N(β$_1$) = 2.432.10^{10}

Structure	1	2	3	4	5	6	7
$\psi_{k,N}(\beta_1,d)$	49.12	48.21	48.78	27.35	47.12	46.25	44.75
$\psi_{k,1}(\beta_1,0)$	51.6	48.7	1.7	7.63	47,6	46,4	1

For the indoor air $\psi_i(\beta_1)$ = 41.53

Total heat loss at the unit temperature difference 1/R$_o$ = 27 W/K.

The contribution of the term corresponding the first eigenvalue to the exact solution of indoor air temperature at the steady state and the constant unit diffuse solar radiation flow (0,037 K) represents:

$$\frac{\Psi_i(\beta_1)\sum_{\substack{k=1 \\ k\neq 3,4}}^{7}\Psi_{k,N}(\beta_1,d)\cdot\varphi_{4,k}}{\beta_1\cdot N(\beta_1)} = 0.034K$$

which is 92% of the exact value.

The contribution of the term corresponding to the first eigenvalue to the exact solution of indoor air temperature suddenly after the time τ = 0 during the cooling with unit initial temperature:

$$\frac{\Psi_i(\beta_1)\bar{t}(\beta_1,0)}{N(\beta_1)} = 0.88K$$

Equivalent thermal parameter	Analysis	Experiment
$1/R_o$ [W/K]	27	33
$1/R_i$ [W/K]	198	210
C [J/K]	11335013	13016772

The identification of the equivalent thermal parameters of three different buildings confirmed the correctness of the application of simplified model. The values obtained by the experiment, compared with the values obtained analytically were in a good agreement.

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6. CONCLUSIONS

An analytical method using the integral-transform technique was applied to solve the non-steady heat transfer in buildings.
The solution was used at the identification of equivalent building thermal parameters.
The achieved results were compared with the results of experimental identification.

REFERENCES

[1] May, W.B. (1982) Equivalent thermal parameters from measured data. Building Res. Practice 10, 348-361.
[2] Ozisik, M.N. (1980) Heat conduction, John Viley and Sons, New York.
[3] Sicard, J., Bacot, P, Neveu, A. (1985) Analyse modale des échanges thermique dans le batiment. Int. J. Heat Mass Transfer 28, 111-123.
[4] Matiasovsky, P. (1992) The equivalent thermal parameters, their analytical and experimental identification. Solar Energy Materials and Solar Cells 27, 119-126.
[5] Matiasovsky, P. (1996) Daily characteristics of air temperature and solar irradiation / input data for modelling of thermal behaviour of buildings. Atmospheric Environment 30, 537-542.