THERMOPHYSICAL SENSORS: THEORY AND APPLICATION OF THE HOT BALL

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Abstract
The paper deals with the theory and the application of the hot ball sensor. The sensor in a form of a small ball generates heat and simultaneously measures the temperature response. The thermal conductivity is to be determined from the measured signal. The sensor has been applied for monitoring of the moisture in the wall of the Cathedral Duomo in Florence and the setting of cement paste.

Keywords:
transient methods, hot ball method, moisture monitoring, concrete setting monitoring, cement paste

1 INTRODUCTION

Recently a new class of the dynamic methods – transient methods for measuring thermophysical properties started to be spread in research laboratories as well as in technology. The principal difference between the classic and the transient methods lies in the variability of the specimen size, the measuring time and in the number of measured parameters. Transient methods need significantly shorter time for measurement, posses high variability in the specimen size, some of the transient methods can determine the specific heat, thermal diffusivity and the thermal conductivity within a single measurement [1, 2]. The technique of the transient methods has initiated construction of a range of new innovative laboratory instruments [3, 4, 5, 6, 7, 8, 9]. Improvements in methodology of the transient methods and use of recent electronic elements allow construction of portable instruments and the monitoring systems that significantly simplified operation [6, 10]. Construction of such devices evoked a look for suitable sensors that give information on thermophysical properties of the tested object. Up to now a principle of the hot wire (Needle probe [5]), hot strip and hot bridge [3], hot disc (Gustafsson probe [4]) and the plane heat source [1] are most often used. Recently, it was published a principle of a hot ball sensor in two components configuration. i.e. the heat source and the thermometer are fixed apart each other [11].

The present paper deals with the hot ball sensor in a single component configuration i.e. when heat source and the thermometer are unified in a single component. Three models are presented namely the steady state model considering a hot ball made of real material in an infinitive medium, and the transient models, the model of the empty ball in an infinitive medium and the model of a ball made of high thermal conductivity material that is placed, again, in an infinitive medium. While for the former model the conditions are sought where hot ball body has a negligible influence on the thermal conductivity determination. The latter two models give time criterion to reach the steady state conditions, i.e. the time period when the transient vanishes. The model of the empty ball in the infinitive medium represents the ideal model. Calculations are compared with the real construction of the hot ball considering real material properties.

2 THEORY OF THE HOT BALL SENSOR

The working relation of the hot ball sensor is based on an ideal model. The ideal model assumes a constant heat flux from the empty sphere of radius r_b into the infinitive

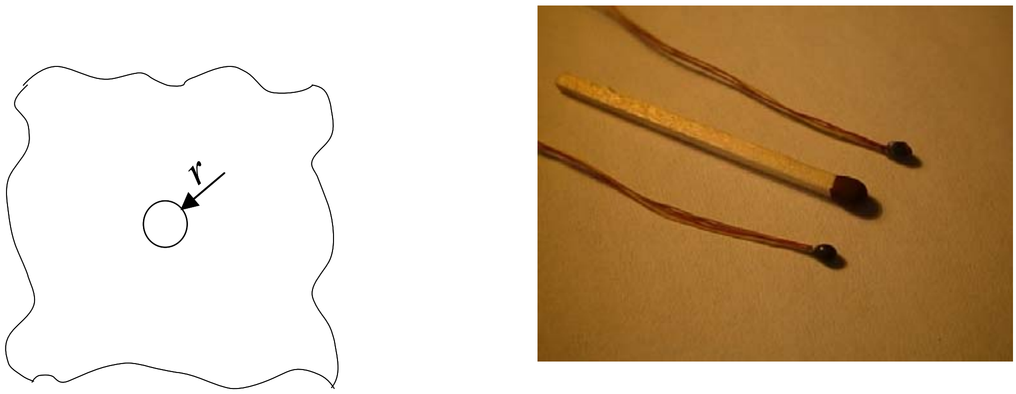

Fig. 1.: Model of the hot ball (left) and its realization (right)

medium that starts to be delivered for times t > 0 (see Fig. 4 left). Then the temperature distribution within the medium is characterized by function [12]

$$T(r,t) = \frac{q}{4\pi\lambda r^2}\left\{\sqrt{\frac{at}{\pi}}\left(1 - e^{-\frac{r^2}{at}}\right) + r \cdot erfc\left(\frac{r}{\sqrt{at}}\right)\right\} \tag{1}$$

where erfc(x) is error function defined by $erfc(x) = \frac{2}{\pi}\int_0^x \left(-\zeta^2\right) d\zeta$ and λ and a are thermal conductivity and thermal diffusivity of the surrounding medium, respectively. Function (1) for long time approximation gives a working relation of the measuring method based on the hot ball sensor

$$\lambda = \frac{q}{4\pi r_b T_m\left(t \to \infty\right)} \tag{2}$$

where T_m is maximal temperature that is reached in long time limit at the surface of the empty sphere with radius r_b.

The measuring method based on function (1) belongs, in fact, among the class of transient ones. However, the heat sources of spherical symmetry posses a special feature that for long times yields the steady state regime and this moment is utilized for measuring the thermal conductivity. To fulfill the conditions valid for use of the relation (2) one needs:

- to construct a hot ball that is composed of real materials
- to find the time period in which the steady state regime is reached

Two different models will be analyzed to find the criteria for using the relation (2):

Heat capacity model. The heat ball has to compose of parts that on one side generates constant heat and on the other side measures the temperature response. Assuming that the hot ball is a perfect conductor, the measured temperature can be ascribed to the surface temperature of the hot ball as it requires the relation (2). Such hot ball has its own heat capacity that causes a deviation from the ideal model characterized by relation (1). In addition a contact thermal resistance 1/H between the hot ball and the medium exists. A model that includes the heat capacity of the hot ball Mc^* and the contact thermal resistance $1/H$ is characterized by relation

$$T(t) = \frac{q}{4\pi\lambda r_b}\left[\frac{1+r_b h}{r_b h} - \left(\frac{r_b}{\sqrt{\pi a t}}\right) - \frac{r_b^2\left[2+r_b h(2-f)\right]}{2hf\pi^2\sqrt{at}}\right] \qquad (3)$$

where

$$f = 4\pi r_b^3 \rho \frac{c}{Mc^*}$$

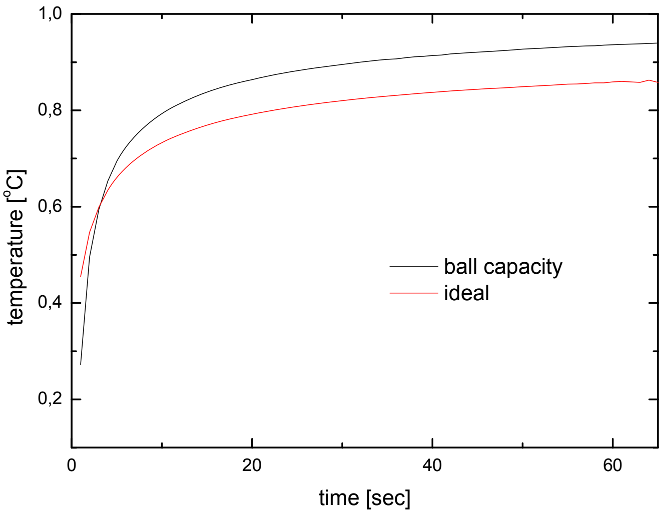

Fig. 2.: Temperature responses for ideal model (relation (2)) and the real one (relation (3)).

and $h = H/\lambda$. The heat capacity of the hot ball Mc^* and the contact thermal resistance $1/H$ disturbs the transient and thus it influence the measuring process. Assuming that parameters values of the hot ball are the following: heat output q = 6 mW, radius r_b = 1 mm, mass M = 0.005 kg, specific heat c^* = 1000 J kg^{-1} K^{-1}, contact thermal conductance H = 6000 W m^{-2} and the tested materials: density ρ = 1000 kg m^{-3}, thermal conductivity λ = 0.5 W m^{-1} K^{-1} and the thermal diffusivity a = 0.5 mm^2 sec^{-1} one obtains data shown in Fig. 2.

An ideal temperature response is plotted in Fig. 2 to find difference between the ideal model using relation (2) and the real one using relation (3). The same thermophysical properties of the tested materials are used in both cases, i.e. density ρ = 1000 kg m^{-3}, thermal conductivity λ = 0.5 W m^{-1} K^{-1} and the thermal diffusivity a = 0.5 mm^2 sec^{-1}. A deviation from the ideal model exists predominantly due to contact thermal resistance $1/H$. A negligible influence of the heat capacity of the hot ball has been found in comparison to the thermal contact resistance considering the shapes of transient temperatures.

Steady state model. The previous analysis was based on the assumption that the hot ball is a perfect heat conductor. This is far from real situation when materials in a broad range of the thermal conductivities are tested. Therefore we look for criterion of the hot ball use considering thermal conductivity range of tested materials. A solution of the partial differential relation will be sought where hot ball and the surrounding material is composed of different materials. The function has form [12]

$$T(r) = q\frac{\left[r_b^2 - r^2 + \frac{2}{H}r_b\lambda_0 + 2r_b^2\left(\frac{\lambda_0}{\lambda}\right)\right]}{6\lambda_0} \quad \text{for } r < r_b \tag{4}$$

and

$$T(r) = q\frac{r_b^3}{3r\lambda} \quad \text{for } r > r_b \tag{5}$$

where $1/H$ is the thermal contact resistance, r_b and λ_0 is the hot ball radius and its thermal conductivity, respectively. q represents the heat production of the hot ball volume unit during time unit. For heat power of the hot ball A we have $q = 3A/(4\pi r_b^3)$.

Analysis has been performed considering previous data, i.e. thermal conductivity of surrounding materials to be $\lambda = 0.5$ W m^{-1} K^{-1} and contact thermal conductance $H = 6000$ W m^{-2}. The radius of the hot ball is to be 0.001 m. The results of the analysis are shown in Fig. 3.

Two cases are shown, namely a hot ball made of material having thermal conductivity $\lambda_0 =$ 0.5 W m^{-1} K^{-1} and that one of 2 W m^{-1} K^{-1}. It should be stressed that real hot ball assumes a negligible temperature gradient within the ball body. Then the hot ball material should have thermal conductivity above $\lambda_0 > 2$ W m^{-1} K^{-1} providing that surrounding material has thermal conductivity below $\lambda < 0.5$ W m^{-1} K^{-1}. Analysis has shown that the criteria H > 10 000 W m^{-2} and $\lambda_0/\lambda > 4$ should be fulfilled to obtain reliable data. A small temperature drop $\Delta T = 0.078$°C at the contact surface of the hot ball with the surrounding material was found for thermal contact conductance H = 6000 W m^{-2}. The analysis has shown the thermal contact conductance should reach value $H >$ 10000 W m-^{2} for good measurements (no temperature drop).

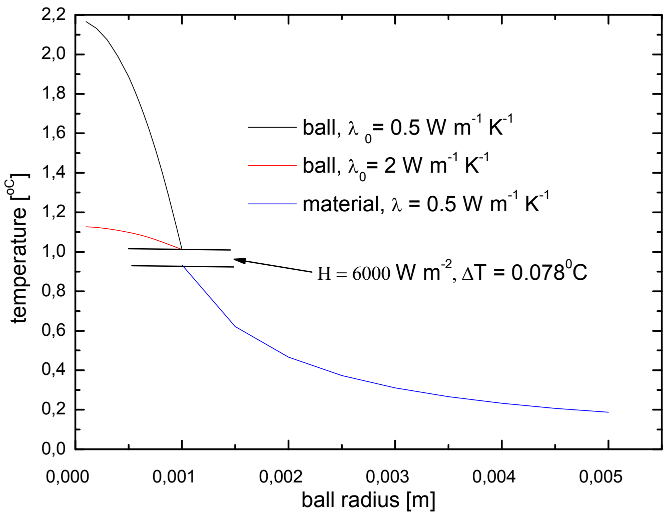

Fig 3.: Temperature distribution within the hot ball and the surrounding materials.

3 VERIFICATION OF THE THEORY

The sensor consists of a heating element and of a thermometer. Both elements are fixed in a ball by epoxy resin (Fig. 1 - right) (patent pending). Diameter of the ball is around 3 mm. The RTM 1.01 instrument was used for measurements. The phenolic foam, aerated concrete, calcium silicate and Sander sandstone was used for verification experiments. Table 2 gives thermophysical data of the tested materials. The period of heating has been adjusted to the thermal conductivity of the tested material and it varies in the range from 100 sec up to 1800 sec.

Table 1.: Materials for testing hot ball.

Material	Thermal conductivity [W m^{-1} K^{-1}]
sandstone	1.9
PMMA	0.19
Aerated concrete	0.155
Calcium silicate	0.097
Phenolic foam	0.06

The hot ball sensor is in principle absolute method for measuring thermal conductivity providing that evaluation method fulfills assumptions given by theory. Fig. 5. gives an overview on reliability of the sensor. Data represent radius of the hot ball calculated by relation (2) and using table 1.

The hot ball sensor has constant radius within 15% in the thermal conductivity range from 0.04 up to 0.4 W $m^{-1}K^{-1}$. However the radius deviates from the real one. The radius of the hot ball has been measured to be 1.5 mm. The sensor is constructed for application in environmental and industrial conditions. Specimen size up to 1 cm^3 is required when thermal conductivity of material has to be determined.

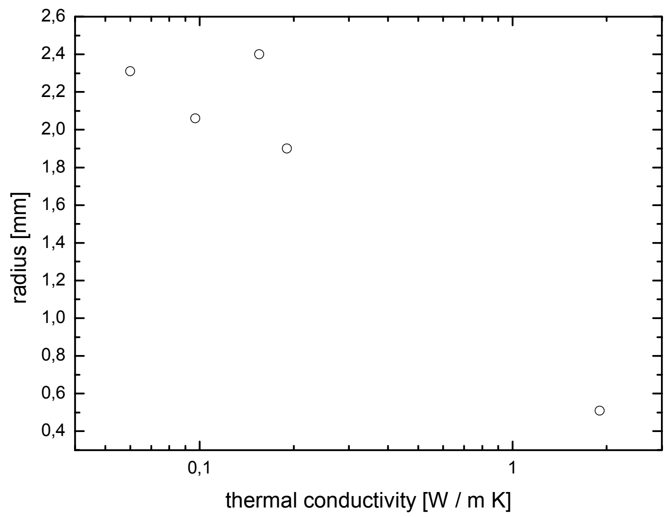

Fig. 5.: Reliability of the hot ball sensor tested by its radius.

4 APPLICATION OF THE HOT BALL

The hot ball sensor can be used in the monitoring regime providing that appropriate instrumentation is constructed. An inexpensive instrument based on the logger in connection with the microprocessor has been constructed (Transient MS s.r.o.[6]). More details on the measuring regime can be found elsewhere [14]. The instrument RTM 1.01 in connection with the hot ball has been used for monitoring the moisture regime in the wall and for monitoring of the stiffening the cement paste. A simple relation is used between the moisture content in the porous structure and its thermal conductivity [13]. The stiffening of the cement paste significantly influence its thermal conductivity predominantly due to increase of the velocity of the acoustic waves.

Monitoring of the moisture in the wall. The hot ball sensor in connection with the instrument RTM 1.01 can be used for monitoring of moisture in the wall. This type of experiment has been realized on the Cathedral Duomo in Florencia (Fig. 6 - left). The sensor fitting at the marble plate is shown in Fig. 6 – right. Information on moisture concerns the sensor surrounding through value of the thermal conductivity. The sensor was fixed at the surface of the marble plate by the help of a block made of the same material just below the cupola. The block was painted by a watertight varnish. Thus moisture has diffused to the sensor surrounding in the surface layer. The RTM 1.01 was set to perform measurements with repetition of 30 minutes. The heat power of 5 mW during 30 sec was used for measuring. Data on temperature and the thermal conductivity collected in period of February 8 – March 14, 2005 are shown in Fig. 7. A standard variation of wall temperature can be found due to day – night alternation. No changes of moisture can be found for dry period. However variation of moisture was found due to precipitation. As we have mentioned that information on thermal conductivity – moisture is taken from the sensor surrounding then level of moisture was strongly influenced by temperature. The temperature increase induced growth of the water diffusion rate. Thus moisture has increased in the marble volume below the block (see Fig. 6 – right). Diffusion process is slow in comparison with the measuring process that takes about 120 sec. Thus the temperature and the moisture are measured in quasi-equilibrium state.

Fig.6.: Cupola of the Duomo Cathedral (left) and the fixing of the hot ball and the instrument RTM 1.01.

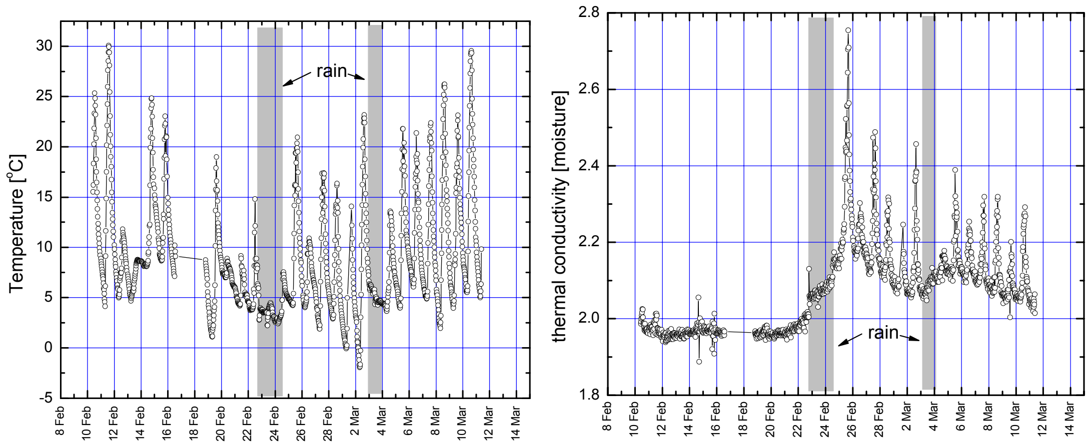

Fig. 7.: Temperature (left) and the moisture (right) in the marble plate of the Cathedral Duomo.

Stiffening of the cement paste. The cement paste has been prepared from the mixture of a portland cement CEM I 42.5R (HOLCIM plant) with the water-to-cement ratio w/c of 0.29. A steel tube in length of 50 mm and in diameter of 30 mm was filled with mixture. The hot ball sensor was fixed in the centre of the tube and the monitoring instrument RTM 1.01 (Transient M S s.r.o.[6]) was connected with the sensor. The setting was measured as a function of time.

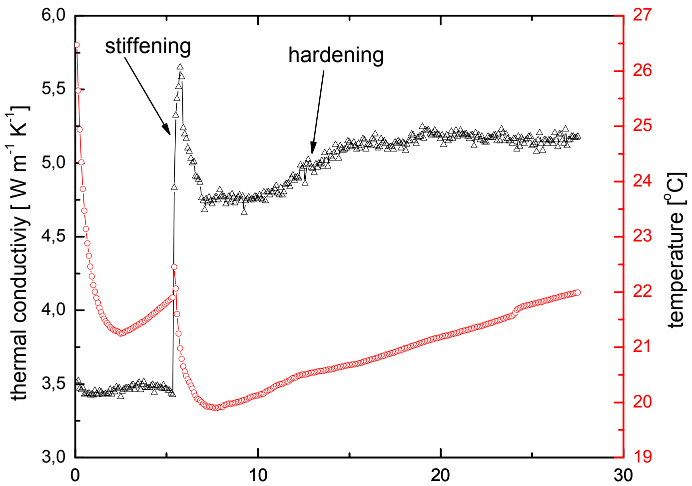

Fig. 6.: Thermal conductivity (right) and the temperature (left) as a function of time during the setting of the cement paste.

The RTM 1.01 was set to perform measurements with the repetition of 10 minutes. The experiment was performed at 25^{o}C and 30% humidity of the surroundings. The heat power of 8 mW during 30 sec was used for measuring. Experimental data are shown in Fig. 6. An anomaly of the temperature was found during the stiffening process while no temperature variation exists during hardening. The stiffening process is connected with strong increase of the thermal conductivity. This is due to increase of the strength of the material during the stiffening that induces the growth of the acoustic velocities of the phonons. Lowering the thermal conductivity is due to drying of the material due to hydration process. A hardening can be found after 12 hours. The whole process finished within 20 hours however, the hydration process has been not finished due to deficiency of water.

5 CONCLUSIONS

Theory of the hot ball sensor and its application has been presented. Three models were analyzed. The ideal model of the empty ball in an infinitive medium, the ball made of a perfect conductor in an infinitive medium and the steady state measuring regime when both the hot ball and the surrounding medium are made of real materials. The contact thermal resistance is the most distorting factors that influences the temperature response. The criteria $H > 10\ 000$ W m^{-2} and $\lambda_0/\lambda > 4$ has to be fulfilled to obtain reliable data. The theoretical time period for reaching the steady state regime agree with the experiment considering the material having thermal conductivity around $\lambda = 0.5$ W m^{-1}K^{-1}. The sensor has been applied for monitoring the moisture in the wall of the Cathedral Duomo in Florence. Variations in temperature and the moisture were found depending on day – night period and the dry and rain period. The sensor was applied for monitoring the setting process of the cement paste. A clear stiffening and hardening process can be recognized.

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