APPLICATION OF INVERSE METHOD TO THE DETERMINATION OF HEAT TRANSFER COEFFICIENT FOR POROUS MATERIAL

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Abstract

The approach based on the solution of an inverse heat transfer problem and surface temperature measurements is used for determination the temperature-dependent heat transfer coefficient of porous building material and for the experiment design. The transient surface temperature distributions are monitored by the infrared camera. The Levenberg-Marquardt procedure of minimization of the least-squares norm is applied for a solution of the presented parameter estimation problem. The results of the application of this method to a real temperature field showed its suitability for practical engineering calculations.

Key words: heat transfer coefficient, inverse thermal problem, infrared camera

1 Introduction

The heat transfer coefficient given by the Newton`s law of cooling of porous materials exposed to normal climatic conditions can be determined by standard laboratory techniques. However, a current research in determining the building porous material thermal properties indicates the unsuitable nature of the standard measuring methods, Shin et al [1].
In order to determine the heat transfer coefficient of building porous materials as a function of temperature, the inverse heat transfer problem should be solved. The inverse methods are well known, in particular in the works [2, 3].
The proposed procedure combines the accurate measurements of surface temperature and the data processing with the inverse thermal problem by utilizing the Levenberg – Marquardt method [3] in order to determine the temperature dependence of heat transfer coefficient $h(T)$. The temperature interval is considered as the piece-wise continuous approximation, that is, the inverse thermal problem can be seen as a parameter estimation. Heat transfer coefficient $h(T)$ is represented by the vector $\boldsymbol{h} = (h_1, h_2, ..., h_N)$ as the estimated parameter.

The results of ill-posed inverse problem depend on the number of accurate temperature measurements, that the infrared camera is enabled to acquire. This generally makes the approach fairly stable and accurate.
The D-optimum approach [3] is used for the experiment design - the heating and final experimental time period.

2 Direct problem

The physical problem involves a concrete cylindrical geometry sample, which is mounted vertically and heated from the bottom base by a surface heater. The sample is embedded in the chamber with a constant temperature. The experimental setup ensures axis-symmetrical cooling conditions of 2D transient heat transport in the specimen. The transient temperature distribution on the surface of the sample is measured by the infrared camera (see Fig.1).

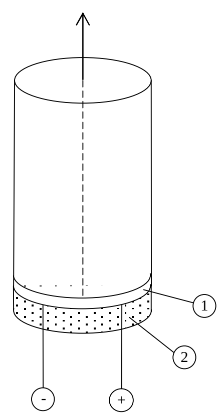

Fig.1 Setup of the experiment 1- heater, 2- insulating basement

The governing equation for the above axis-symmetrical thermal conditions, for the material which thermal conductivity $\lambda(T)$ is temperature dependent and can be written as

$$div\left[\lambda(T)\nabla T\right] = \rho\, c\, \frac{\partial T}{\partial t} \tag{1}$$

where: $T = T(r, z, t)$ is the temperature field, r and z are radius and height of specimen, respectively, ρ and c are density and specific heat.
Heat transfer along the surfaces of the specimen is defined by the boundary conditions, which are expressed as follows

$$-\lambda(T)\frac{\partial T}{\partial z} = q \qquad r \le R \quad \text{and} \quad z = 0 \tag{2}$$

$$-\lambda(T)\frac{\partial T}{\partial r} = h_v(T)(T - T_e) \qquad r = R \quad \text{and} \quad 0 \le z \le H \qquad (3)$$

$$-\lambda(T)\frac{\partial T}{\partial z} = h_h(T)(T - T_e) \qquad r \le R \quad \text{and} \quad z = H \qquad (4)$$

where: q is the heating flux, $h_v(T)(T - T_e)$ and $h_h(T)(T - T_e)$ represent the heat exchanged with the ambient air, $h_v(T), h_h(T)$ are the heat transfer coefficients dependent on temperature, which control the cooling process along side and top surfaces, respectively, T_e is the ambient temperature.

The initial condition are expressed as follows

$$T(r, z, t) = T_e \qquad r \le R \qquad \text{and} \qquad 0 \le z \le H \qquad (5)$$

It is considered that the temperature range of interest in building applications is 20 – 80º C. In the solution of the formulated direct problem, we employ a centric-difference scheme.

3 Inverse problem

For the inverse problem analysed here, the heat transfer coefficient $h(T)$ is regarded as an unknown quantity. The aim of the inverse analysis is to identify it. The whole temperature range is divided into a certain number of sub-ranges within which the heat transfer coefficient is modeled as a piece-wise linear function. This practically means that the function $h(T)$ is modeled as the vector $\boldsymbol{h} = (h_1, h_2, ..., h_N)$, where h_n represents a value of heat transfer coefficient at a selected temperature T_n, $n = 1, ...N$. For the determination of such parameters, we consider the transient temperature measurements T_{ij}^m taken at the locations along the side surface x_j, $j = 1,..., M$. The subscript i refers to the time at which the measurements are taken, that is t_i for $i = 1,..., I$. The temperature measurements may contain random errors, but all the other quantities appearing in the formulation of the direct problem are supposed be known exactly. By assuming the normally distributed random errors, with the constant standard deviation and zero mean, the solution of the inverse analysis leads to the optimization of the ordinary least-squares norm, which can be written as

$$S(\boldsymbol{h}) = \left[\mathbf{T}^m - \mathbf{T}^c(\mathbf{h})\right]^{\mathrm{T}} \left[\mathbf{T}^m - \mathbf{T}^c(\mathbf{h})\right] \qquad (6)$$

and which reaches the minimum among all admissible vectors $\boldsymbol{h}$;
where $\boldsymbol{h} = (h_1, h_2, ..., h_N)$ denotes the vector of unknown parameters. The superscript T denotes transpose and $\left[\mathbf{T}^m - \mathbf{T}^c(\mathbf{h})\right]^{\mathrm{T}}$ is given by

$$\left[\mathbf{T}^m - \mathbf{T}^c(\mathbf{h})\right]^{\mathrm{T}} \equiv \left[\left(\vec{T}_1^m - \vec{T}_1^c\right), \left(\vec{T}_2^m - \vec{T}_2^c\right), ..., \left(\vec{T}_I^m - \vec{T}_I^c\right)\right]$$

where $\left(\vec{T}_i^m - \vec{T}_i^c\right)$ is a row vector containing the differences between the measured and estimated temperatures at the measurement points x_j, $j = 1,...,M$, at time t_i, that is: $\left(\vec{T}_i^m - \vec{T}_i^c\right) = \left[T_{i1}^m - T_{i1}^c, T_{i2}^m - T_{i2}^c, ..., T_{iM}^m - T_{iM}^c\right]$

The temperatures T_{ij}^c are calculated from the solution of the direct problem (at the positions and time where temperatures T_{ij}^m are measured) by using estimates for the unknown parameters h_n, $n = 1,2,...,N$.

The minimizing procedure, in which components of the vector ***h*** are updated, is realized with Levenberg–Marquardt method. The iterative procedure is given by

$$\mathbf{h}^{k+1} = \mathbf{h}^k \left[\left(\mathbf{J}^k\right)^{\mathrm{T}} \mathbf{J}^k + \mu^k \Omega^k\right]^{-1} \left(\mathbf{J}^k\right)^{\mathrm{T}} \left[\mathbf{T}^m - \mathbf{T}^c\left(\mathbf{h}^k\right)\right] \quad (7)$$

where $\mathbf{J}^k$ is the sensitivity matrix, μ^k is a positive scalar (damping parameter), Ω^k is a diagonal matrix and the superscript k denotes the iteration number.

The matrix term $\mu^k \Omega^k$ damps oscillations and instabilities due to the ill-conditioned character of the problem by making its components large compared to those of $\mathbf{J}^{\mathrm{T}}\mathbf{J}$. The damping parameter is reduced as the iteration procedure advances to the solution and it increased if the errors inherent to the measured data are amplified generating instabilities on the solution. The stopping criterion is used as follows $\left\|\mathbf{h}^{k+1} - \mathbf{h}^k\right\| < \varepsilon$, where ε is the desired tolerance.

The sensitivity matrix $\mathbf{J}$ is defined as

$$\mathbf{J}(\mathbf{h}) \equiv \left[\frac{\partial \mathbf{T}^{\mathrm{T}}(\mathbf{h})}{\partial \mathbf{h}}\right]^{\mathrm{T}} \quad (8)$$

The elements of the sensitivity matrix are sensitivity coefficients, which are defined as the first derivative of the calculated temperatures with respect to the assumed and then identified input data h_n, $n = 1,2,...,N$.

The sensitivity coefficients are required to be large in a magnitude, so that the estimated parameters are not very sensitive to the measurement errors. In order to have the matrix $\mathbf{J}^{\mathrm{T}}\mathbf{J}$ invertible the determinant of $\mathbf{J}^{\mathrm{T}}\mathbf{J}$ cannot be zero or very small. Such a requirement over the determinant of $\mathbf{J}^{\mathrm{T}}\mathbf{J}$ is better understood by taking into account a statistical analysis. The confidence region for the estimated parameters $h_1, h_2, ..., h_N$ is computed from

$$\left(\hat{\mathbf{h}} - \mathbf{h}\right)^{\mathrm{T}} \mathbf{V}^{-1} \left(\hat{\mathbf{h}} - \mathbf{h}\right) \le \chi^2 \quad (9)$$

where: χ^2 is the chi-square distribution, and $\mathbf{V}$ is the covariance matrix of the estimated parameters $h_1, h_2, ..., h_N$ given by

$$\mathbf{V} = \left(\mathbf{J}^{\mathrm{T}}\mathbf{J}\right)^{-1} \sigma^2 \quad (10)$$

where σ is the standard deviation.

The design of an optimum experiment basically consists in examining a priori some kind of measure of the accuracy of the estimated quantities in order to choose experimental variables (in this paper the heating time), so that minimum variance estimates are obtained. The optimum experiment is designed by minimizing the hypervolume of the confidence region. The minimization is obtained by maximizing the determinant of $\mathbf{V}^{-1}$, in the D-optimum approach [3]. Since the covariance matrix $\mathbf{V}$ is given (10), we can the design the optimum experiment by maximizing the determinant of the matrix $\mathbf{J}^{\mathrm{T}}\mathbf{J}$, also refered to as the Fisher information matrix. For a case involving a large but fixed number of transient measurements of M sensors, each element $\mathbf{F}_{m,n}$, $m,n = 1,2,...,N$ of the matrix $\mathbf{F} \equiv \mathbf{J}^{\mathrm{T}}\mathbf{J}$ is given by

$$\mathrm{F}_{m,n} = \frac{1}{M \cdot \tau_f} \sum_{j=1}^{M} \sum_{i=1}^{I} \left(h_m \frac{\partial T_{ji}}{\partial h_m} \right) \left(h_n \frac{\partial T_{ji}}{\partial h_n} \right) \tag{11}$$

where τ_f is the final experimental time.

The effectiveness of the used iteration process is measured by the residual

$$RMS = \sqrt{\frac{1}{M \cdot I} \sum_{i=1}^{I} \sum_{j=1}^{M} \left(T_{ij}^{m} - T_{ij}^{c}(\mathbf{h}) \right)^2} \tag{12}$$

4 Results

In this work, the temperature dependent heat transfer coefficient was determined from the experimental set-up with transient thermal conditions, as we mentioned in section 2. The sample has the radius $0.025\,m$, the height $0.08\,m$, the temperature dependent thermal conductivity is $(0.82 + 0.015 \cdot T)\ W.m^{-1}.K^{-1}$ for the temperature interval $(20-60)\,°C$, the heat flux supplied by the heater is $380\,W.m^{-2}$, the temperature in the chamber is $20\,°C$. The surface temperature distribution was carried out by NEC

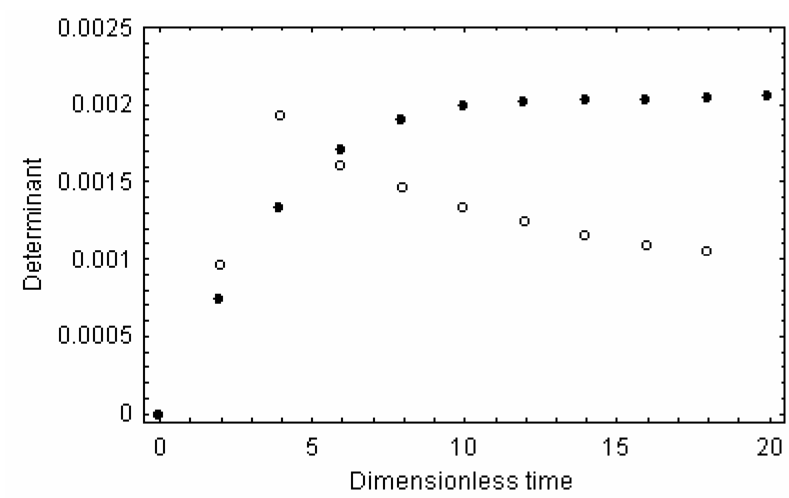

Fig. 2 Transient variation of $\det(\mathbf{F})$ for heating time $\tau_h = 3$ (o) and for continuous heating (•)

TH7102MX infrared camera. Such images guarantee that measurement errors generally do not exceed $0.2K$. As explained in the section 3, we examine the experimental heating time, so that the minimum variance estimated quantities are obtained. Fig. 2 presents the time variation of the determinant of the matrix $\mathbf{F}$, the elements of which are given by (10). Fig. 2 was obtained for two heating times. An analysis of Fig.2 reveals the fact that the maximum value of $\det(\mathbf{F})$ for the heating time $\tau_h = 3$ is about the same as the one obtained with the continuous heating. But, the maximum value of $\det(\mathbf{F})$ for $\tau_h = 3$ occurs at the final experimental time of $\tau_f = 4{,}2$, instead of $\tau_f = 9{,}5$ for continuous heating. Therefore the use $\tau_h = 3$ and $\tau_f = 4{,}2$ may result in estimates as accurate as those obtained with continuous heating, but in less then half of the experimental duration.

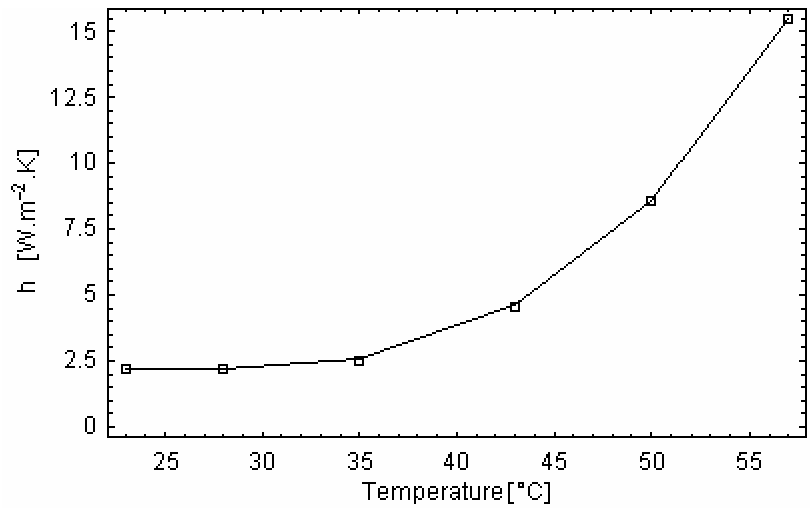

Fig.3. Temperature dependence of heat transfer coefficient.

Fig.3 demonstrates the results obtained for the heat transfer coefficient along the side of a specimen as the temperature dependence. It is important to stress that the final results of inverse analysis do not depend on the initial guesses. The initial guess affects the number of iterations but not the values to which the identified quantities converge. Fig.4 presents the convergence of iteration process for the identification of three components of heat transfer coefficient.

Conclusions

In this paper the procedure of determining of the heat transfer coefficient of concrete materials, based on the transient temperature measurements using an infrared camera is proposed. The collected temperature measurements are processed through the inverse thermal modeling software which utilizes an appropriate model of heat transfer

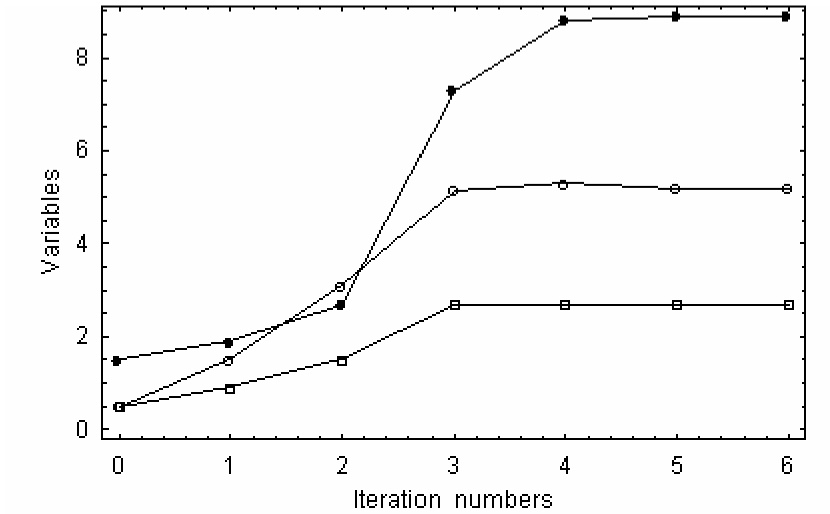

Fig.4. Convergence of the iteration process for: □ $h(32°C)$, o $h(43°C)$, • $h(52°C)$

phenomena and identifies the heat transfer coefficient as the temperature function. The extensive calculations showed that the iterative process converges to the values quite close to accurate ones. An analysis of the determinant of the information matrix shows that more accurate estimates can be obtained by using a heating time smaller than the final experimental time. The resolution of infra-red camera measured temperatures with the accuracy of $0.2K$ guarantees a good accuracy of identification.

Acknowledgment

The financial assistance of the Grant Agency of Sciences of the Slovak Republic, within grant No.1/8334/01 and Slovak Science and Technology Assistance Agency under number APVT-51-030704 is gratefully acknowledged herewith.

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