NONLINEAR PROBLEM OF INVERSE DETERMINATION OF HEAT COEFFICIENT

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Abstract

The objective of this paper is to determine temperature dependent heat transfer coefficient for the characterization of porous materials in the range 20 – 60 º C, using the approach based on the solution of the inverse heat transfer problem and results of measuring boundary conditions. The surface temperature distribution is measured using the infrared camera.

Key words: heat transfer coeficient, inverse method, infrared camera

1 Introduction

The heat transfer coefficient - quantity controlling heat transfer process is given Newton`s law of cooling, h, of porous materials exposed to normal climatic conditions can be determined standard laboratory techniques. However, current research in determining the construction materials thermomechanical properties indicates the unsuitable nature of standard measuring methods, Shin et al [1].

In order to determine the thermal characteristics of structural, thermal-protective and thermal-insulating materials as a function of temperature, the inverse heat transfer problem should be solved. New methodology combines accurate measurements of thermal quantities, which can be experimentally observable in real conditions and accurate data processing, based on the solution of heat transfer problems.

In the present paper, the method for estimating heat transfer coefficient is carried out for building porous material. For such material the goal is to estimate the characteristic as temperature function by using results of measuring boundary conditions and surface temperature of body under consideration. The proposed procedure of determining of heat transfer coefficient consist of heating a cylindrical geometry sample by a surface heating element and measuring (after reaching a steady-state) temperatures along side of the specimen using a infra-red camera. Temperature measurements are then processed through the inverse thermal modeling software, which identifies unknown heat transfer coefficient occurring in the model. It has to be stressed, that a infra-red camera used in the experiment allows to utilize a large amount of temperature measurement. This generally makes the approach fairly stable and accurate.

2 Experimental setup

Concrete cylinder geometry sample is mounted vertically and heated from the bottom base by surface heater, which is fed a known electrical power and provides constant thermal energy. The sample is embedded in chamber with constant temperature. The experimental setup ensures axis-symmetric cooling conditions of 2D steady–state heat transport in the specimen. The surface temperature distribution of the specimen is measured infrared camera, see Fig.1.

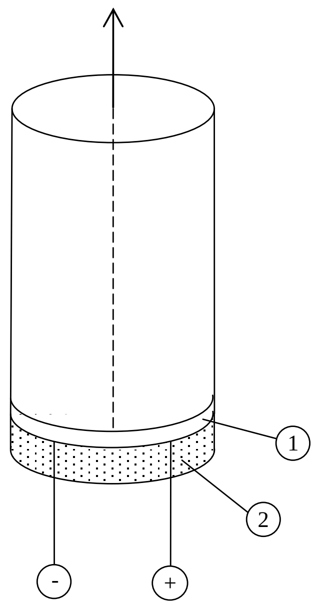

Fig.1 Setup of the experiment 1- heater, 2- insulating basement

3 Mathematical model of 2D steady state heat transfer

The governing equation for above axis-symmetric steady-state thermal conditions, for material which thermal conductivity $\lambda(T)$ is temperature dependent, can be written as

$$div[\lambda(T)\nabla T] = 0 \qquad (1)$$

where $T(r,z)$ is temperature field, r and z are radius and height of specimen, respectively, ρ and c are density and specific heat.

Heat transfer along the surfaces of the specimen is defined by boundary conditions, which are expressed as follows

$$-\lambda(T)\frac{\partial T}{\partial z} = q \qquad r \le R \quad \text{and} \quad z = 0 \qquad (2)$$

$$-\lambda(T)\frac{\partial T}{\partial r} = h_v(T)(T - T_e) \qquad r = R \quad \text{and} \quad 0 \le z \le H \tag{3}$$

$$-\lambda(T)\frac{\partial T}{\partial z} = h_h(T)(T - T_e) \qquad r \le R \quad \text{and} \quad z = H \tag{4}$$

where q is the heating flux, $h_v(T)(T - T_e)$ and $h_h(T)(T - T_e)$ represent the heat exchanged with the ambient air, $h_v(T), h_h(T)$ are the heat transfer coefficients dependent at temperature, which control cooling process along side and top surfaces, respectively, T_e is the ambient temperature.

Considering the temperature range of interest in building applications is (20 – 60) º C.

4 Inverse determination of heat transfer coefficient

The objective is to determine the heat transfer coefficients with the assumption that everything in the direct heat transfer problem (1) – (4) is known except for one and some temperatures T_i^m where $i = 1,2,...,N_m$, measured at well defined positions along side surface have to be provided. The results of surface temperature measurements are assigned as necessary additional information to solve the inverse problem. In the inverse problem (1) – (4) it is necessary first of all to indicate as a temperature range $|T_{\min}, T_{\max}|$ of the unknown function (heat transfer coefficient), which is general for experiment, and for which the inverse problem analysis has a unique solution. In the solution of the formulated direct problem, we employ a centric-difference scheme.

As already mentioned, it is assumed that heat transfer coefficient varies with temperature. Hence the whole temperature range is divided into certain number of sub-ranges within which particular properties are modeled as pice-wise linear. This practically means that one has to identify small number of parameters which represent heat transfer coefficient values at selected temperatures.

We want to find such a heat transfer coefficient $\boldsymbol{h} = (h_1, h_2, ..., h_{N_h})$, that the objective function [2]

$$S(\boldsymbol{h}) = \sum_{i=1}^{N_m} \frac{1}{\sigma_T^2}\left(T_i^m - T_i^c(\boldsymbol{h})\right)^2 + \sum_{j=1}^{N_h} \frac{1}{\sigma_{h_j}^2}\left(h_j - h_j^0\right)^2 \tag{5}$$

reaches the minimum among all admissible vectors $\boldsymbol{h}$;

where $(h_1, h_2, ..., h_{N_h})$ stand for the heat transfer coefficient values at selected temperatures (respectively locations along site and top surfaces), T_j, with $j = 1,2,...,N_l$; $T_i^c(\boldsymbol{h})$ are calculated temperatures (at the positions where temperatures T_i^m are measured) with estimates for the unknown quantities $\boldsymbol{h}$. The standard deviation σ_T is error associated with the temperature measurement. In processing the results of real experiments there are always errors, depending on a number of reasons. First of all, the errors in the experimentally measured data are both, random and systematic by nature. Random errors in input data are stipulated by a spread of thermal and electrical characteristics in measuring devices, by inaccuracy of their calibration, etc, as a rule, these errors show a large enough value. Systematic errors in the input data are usually connected with inaccuracy in determining the coordinates of positions of T_i^m, with displacement of the specimen during filming survey etc. The second group of errors – the errors of finite–difference approximation of differential operator in initial problem and round-of errors in the computer. Besides, there are errors because of uncertainties in the a-priori assigned characteristics of mathematical model (1) – (4), which are determined either through calculations or from a solution of the corresponding inverse problem. The deviation σ_{h_j} is an interval within which each of the parameters h_j is allowed to vary around a priori (i.e., guessed) parameter h_j^0. The function $S(\boldsymbol{h})$ becomes the standard least-squares method when the σ_j are set to infinity, and h_j will be fixed to the value h_j^0. The values h_j in each interval, $[T_j, T_{j+1}]$, are linearly interpolated. The minimizing procedure, in which components of the vector $\boldsymbol{h}$ are updated, is based on the concept of sensitivity coefficient [3]. In order to minimize $S(\boldsymbol{h})$, one writes

$$\frac{\partial S}{\partial h_j} = \sum_{i=1}^{N_m} \frac{-2}{\sigma_T^2}(T_i^m - T_i^c(\boldsymbol{h})) \cdot Z_{ij} + \frac{2}{\sigma_{h_j}^2}(h_j - h_j^0) = 0 \tag{6}$$

where Z_{ij} is the sensitivity coefficient

$$Z_{ij} = \frac{\partial T_i^c(\boldsymbol{h})}{\partial h_j} = \frac{T_i^c\left(h_1,\ldots,h_j + \delta h_j,\ldots,h_{N_h}\right) - T_i^c\left(h_1,\ldots,h_j,\ldots h_{N_h}\right)}{\delta h_j} \tag{7}$$

where δh_j is an a priori variation of the parameter h_j. In the iterative procedure, the calculated temperatures $T_i^c\left(\boldsymbol{h}^{k+1}\right)$ at iteration $(k+1)$ is linearized as follow

$$T_i^c\left(\boldsymbol{h}^{k+1}\right) = T_i^c\left(\boldsymbol{h}^k\right) + \sum_{r=1}^{N_h} Z_{ir} \cdot \Delta h_r \tag{8}$$

It is quite easily verify, that substituting (8) and (9) the system of linear equations is obtained as follow

$$\sum_{r=1}^{N_h}\left(\sum_{i=1}^{N_m} \frac{Z_{ij} Z_{ir}}{\sigma_T^2} + \frac{\delta_{jr}}{\sigma_h^2}\right) \cdot \Delta h_r = \sum_{i=1}^{N_m} \frac{1}{\sigma_T^2}\left(T_i^m - T_i^c\left(\boldsymbol{h}^k\right)\right) \cdot Z_{ij} - \frac{1}{\sigma_h^2}\left(h_j^k - h_j^0\right) \tag{9}$$

where δ_{jr} is the Kroneker symbol. The increments of the parameters $\Delta\boldsymbol{h}$ are found at each iterations as the solution of the system (9).

The identification procedure of the quantities $h_1, h_2, \ldots, h_{N_h}$ is as follows. The components of vector $\boldsymbol{h}$ are initiated to some values $\boldsymbol{h}^0$ and the temperatures $T_i^c\left(\boldsymbol{h}^0\right)$ are calculated from the direct problem. (In this work the centric - difference approximation is utilized.) Each of the parameters h_j^0 is varied by δh_j, temperatures are again calculated and sensitivity coefficients are deduced (each time the vector $\boldsymbol{h}$ is update) using (7). $N_h \times N_h$ system equations (9) is solved to obtain the increments Δh_j and h_j values are updated:

$h_j^{k+1} = h_j^k + \Delta h_j$, the calculation proceeds with next iteration. The procedure is finished, if the maximum relative variation of the parameters $\left|\Delta h_j / h_j\right|$ is smaller than a desired tolerance. This presented modified the least-squares technique is implemented as main program calling the direct CDM heat flow code [2]. The effectiveness of the used iteration process is measured by the residual

$$RMS = \sqrt{\frac{1}{N_m}\sum_{i=1}^{N_m}\left(T_i^m - T_i^c(\boldsymbol{h})\right)^2} \tag{10}$$

5 Results

In this work, the temperature dependent heat transfer coefficient was determined from experimental set-up with steady-state thermal conditions, as we mentioned in section 2. The sample has radius $0.025\,m$, height $0.08\,m$, temperature dependent thermal conductivity is $(0.82 + 0.015 \cdot T)\ W.m^{-1}.K^{-1}$ for temperature interval $(20-60)°C$, the heat flux supplied by heater is $348\,W.m^{-2}$, the temperature in the chamber is $20\,°C$. The surface temperature distribution was carried out by NEC TH7102MX infrared camera. Such images guarantee that measurement errors generally do not exceed $0.2K$. An infrared picture of the specimen surface is shown in Fig.2.

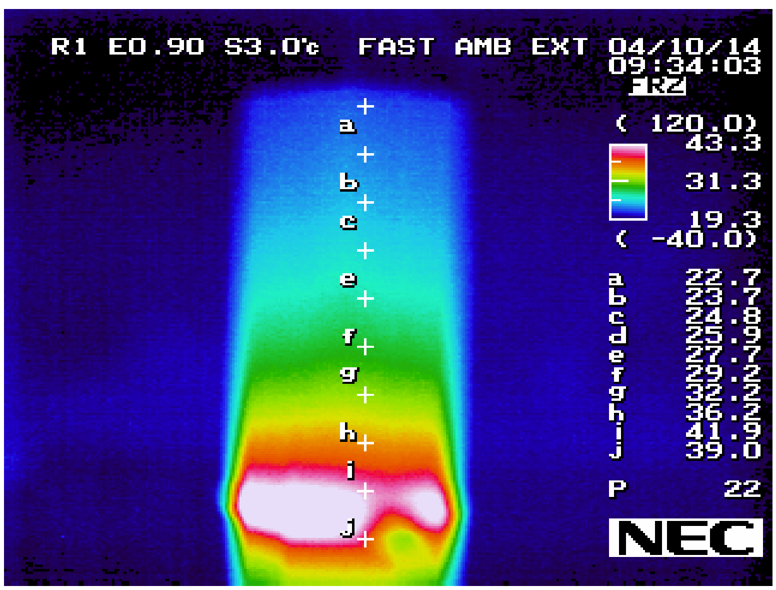

Fig.2 Thermography of surface temperature field

Fig.3 demonstrates results obtained for heat transfer coefficient along side of specimen as temperature dependence.

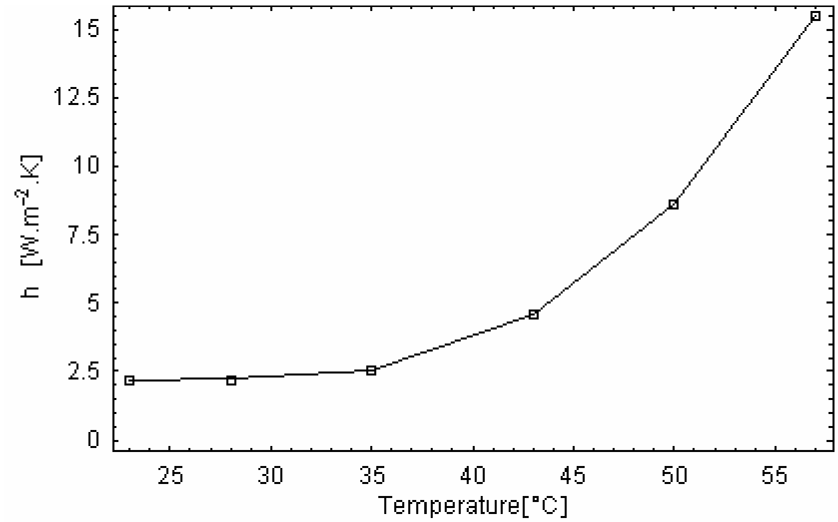

Fig.3 Temperature dependence of heat transfer coefficient

Conclusions

In this paper the procedure of determining of heat transfer coefficient of concrete materials, based on steady-state temperature measurements using an infrared camera is proposed. Collected temperature measurements are processed through the inverse thermal modeling software which utilizing an appropriate model of heat transfer phenomena and identifies heat transfer coefficient as temperature function. Extensive calculations showed that the iterative process converges to values quite close to accurate ones. Resolution infra-red camera measuring temperatures with accuracy of $0.2K$ guarantees a good accuracy of identification.

Acknowledgment

The financial assistance of the Grant Agency of Sciences of the Slovak Republic, within grant No.1/8334/01 is gratefully acknowledged herewith.

Reference

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