ANALYSIS OF VARIOUS EFFECTS ON THE ACCURACY OF MEASURED THERMOPHYSICAL PARAMETERS BY PULSE TRANSIENT METHOD

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Abstract

In the contribution the temperature field in the semi-infinite long sample of the cylindrical form is given. The heat pulse in the form a δ - function and of the finite duration and heat loss from the sample surface are considered.

Key words: thermal conductivity, thermal diffusivity, coefficient of surface heat loss

1 Introduction

For the measurement of the thermophysical parameters there exists many methods but every method is based on the solution of the heat equation. In this paper we are interested in transient method of measuring thermophysical parameters. For the experimentalists it is suitable to use a simple relation for the evaluation of the experimental data. But if we consider all real conditions at which the experiment is done, then the temperature field is expressed by the complicated function. Therefore, our aim is to propose the method how to determine the thermophysical parameters from the experiment if we know the time-dependence of the temperature in a certain place. For realizing this task we have at first to know the temperature field in the sample. The experimental arrangement of the transient method is depicted in Fig.1. We will consider the perfect thermal contact between the heat source and the sample and we will neglect the heat capacity of the heat source. The radius of the planar heat source is smaller than the radius of the sample. Further it will be shown that this choice has some advantage. The sample is infinitely long and has the cylindrical form. The heat loss from the sample surface is also considered. At first we will consider simplified condition of experiment and if it will show that these conditions are not sufficient by the confrontation with experiment, then it will be necessary to consider further conditions.

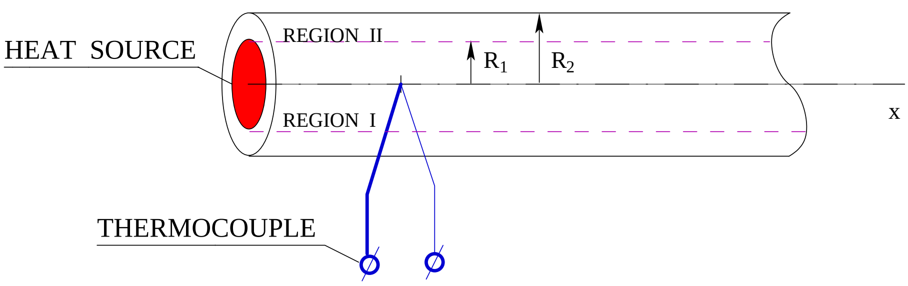

Fig 1 Arrangement of transient method

2 Equations

Due to the various radii of the sample and the heat source we divide the sample in two coaxial cylinders. The heat equations in two regions are the following

$$\frac{\partial T_i}{\partial t} = \frac{\lambda}{c_p \rho}\left(\frac{\partial^2 T_i}{\partial x^2} + \frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r}\frac{\partial T_i}{\partial r}\right) = k\nabla^2 T_i \ , \tag{1}$$

where $i = I, II$, ρ is the density, c_p is the specific heat capacity at constant pressure, λ is the thermal conductivity, $T_I = T_1 - T_0, T_{II} = T_2 - T_0$, T_0 is the temperature of the surroundings, $k = \frac{\lambda}{\rho c_p}$ is the temperature conductivity.

Equation (1) will be solved at the initial conditions:

$$T_I(0,x,r) = T_{II}(0,x,r) = 0 \, . \tag{2}$$

The boundary conditions:

$$T_I(t,\infty,r) = T_{II}(t,\infty,r) = 0 \, , \tag{3}$$

$$-\lambda \frac{\partial T_1}{\partial x}\Big|_{x=0} = qf(t), \qquad -\lambda \frac{\partial T_{I1}}{\partial x}\Big|_{x=0} = 0 \ , \tag{4}$$

where $f(t) = \left\langle \frac{1, \ for \ 0 \le t \le t_0}{0, \ for \ t > t_0} \right.$, q is the heat supplied by the heat source to the sample per unit area and per unit time.

The boundary conditions at $r = R_1$

$$T_I(t,x,R_1) = T_{II}(t,x,R_1) \tag{5}$$

$$-\lambda \frac{\partial T_I}{\partial r}\Big|_{r=R_1} = -\lambda \frac{\partial T_{II}}{\partial r}\Big|_{r=R_1} \tag{6}$$

The boundary condition at $r = R_2$:

$$-\lambda \frac{\partial T_{II}}{\partial r}\Big|_{r=R_2} = \alpha T_{II} \, , \tag{7}$$

where α is the coefficient of surface heat loss. Applying the Laplace's transformation

$$\tilde{T} = \int_0^\infty e^{-pt} T(t) dt$$

and the cosine Fourrier's transformation

$$\tilde{\tilde{T}} = \int_0^\infty cos(ux) \tilde{T} dx$$

on equation (1) and considering the boundary condition (4) we obtain two equations:

$$\left(\frac{p}{k} + u^2 \right) \tilde{\tilde{T}}_I = \frac{q}{\lambda} \frac{1 - e^{-pt_0}}{p} + \frac{\partial^2 \tilde{\tilde{T}}_I}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{\tilde{T}}_I}{\partial r} \tag{8}$$

$$\left(\frac{p}{k} + u^2 \right) \tilde{\tilde{T}}_{II} = \frac{\partial^2 \tilde{\tilde{T}}_{II}}{\partial r^2} + \frac{1}{r} \frac{\partial \tilde{\tilde{T}}_{II}}{\partial r} \tag{9}$$

The solutions of equations (8) and (9) are the following:

$$\tilde{\tilde{T}}_{1I} = \frac{q}{\lambda} \frac{1 - e^{-pt_0}}{p} \frac{1}{\frac{p}{k} + u^2} + C I_0 \left(\sqrt{\frac{p}{k} + u^2}\, r \right) \tag{10}$$

$$\tilde{\tilde{T}}_{II} = B I_0 \left(\sqrt{\frac{p}{k} + u^2}\, r \right) + D K_0 \left(\sqrt{\frac{p}{k} + u^2}\, r \right), \tag{11}$$

where $I_0(x)$ and $K_0(x)$ are Bessel's functions of the imaginary argument.. From the boundary conditions (5), (6) and (7) and from relations (10) and (11) we calculate the constants B,C and D. Introducing these constants into (10) we obtain

$$\tilde{\tilde{T}}_1 = \frac{q}{\lambda} \frac{1 - e^{-pt_0}}{p} \frac{1}{\frac{p}{k} + u^2} + \frac{q}{\lambda} \frac{1 - e^{-pt_0}}{p} \frac{R_1}{\sqrt{\frac{p}{k} + u^2}}\, x$$

$$x \left\{ - K_1 \left(\sqrt{\frac{p}{k} + u^2}\, R_2 \right) + I_1 \left(\sqrt{\frac{p}{k} + u^2} \right) \frac{\sqrt{\frac{p}{k} + u^2}\, K_1 \left(\sqrt{\frac{p}{k} + u^2}\, R_2 \right) - \frac{\alpha}{\lambda} K_1 \left(\sqrt{\frac{p}{k} + u^2}\, R_2 \right)}{- \frac{\alpha}{\lambda} I_0 \left(\sqrt{\frac{p}{k} + u^2}\, R_2 \right) + \sqrt{\frac{p}{k} + u^2}\, I_1 \left(\sqrt{\frac{p}{k} + u^2}\, R_2 \right)} \right\} x$$

$$x I_0 \left(\sqrt{\frac{p}{k} + u^2}\, r \right) \tag{12}$$

After the inverse Laplace's and cosine Fourier's transformation we obtain

1. The heat pulse in the form of δ - function.

In this case we can write

$\lim_{t_0 \to 0} \frac{q t_0}{\lambda} \frac{1 - e^{-p t_0}}{p t_0} = \frac{Q}{\lambda}$, where $Q = q t_0$. In the limit we consider that if $t_0 \to 0,\ q$ increases so that $q t_0$ remain constant.

$$T_I(t,x,r) = 2\frac{Q}{\lambda}\left(\frac{k}{\pi t}\right)^{\frac{1}{2}} e^{-\frac{x^2}{4kt}} - \frac{Q}{\lambda}\left(\frac{k}{\pi t}\right)^{\frac{1}{2}} e^{-\frac{x^2}{4kt}} \sum_{i=1}^{\infty} e^{-\frac{\xi_i^2}{R_2^2}} \frac{\frac{R_1}{R_2}}{\left(\frac{\alpha R_2}{\lambda}\right)^2 + \xi^2} \frac{J_1\left(\xi_i \frac{R_1}{R_2}\right)}{J_0(\xi_i)} J_0\left(\xi_i \frac{r}{R_2}\right) \tag{13}$$

2. The finite duration of the heat pulse

   1. The solution for $0 \le t \le t_0$

$$\begin{aligned} T_I(t,x,r) &= 2\frac{q}{\lambda}\left\{2\left(\frac{kt}{\pi}\right)^{\frac{1}{2}} e^{-\frac{x^2}{4kt}} - x\Phi^*\left(\frac{x}{2\sqrt{kt}}\right)\right\} + \\ &+ \frac{qR_2}{\lambda}\sum_{i=1}^{\infty}\left\{2\cosh\frac{\xi_i}{R_2}x - e^{-\frac{\xi_i}{R_2}x}\left[\Phi\left(\frac{\xi_i}{R_2}\sqrt{kt} - \frac{x}{2\sqrt{kt}}\right) + 2\right] - \right. \\ &\left. - e^{-\frac{\xi_i}{R_2}x}\Phi\left(\frac{\xi_i}{R_2}\sqrt{kt} + \frac{x}{2\sqrt{kt}}\right)\right\} x \frac{\frac{R_1}{R_2}}{\left(\frac{\alpha R_2}{\lambda}\right)^2 + \xi^2} \frac{J_1\left(\xi_i \frac{R_1}{R_2}\right)}{J_0(\xi_i)} J_0\left(\xi_i \frac{r}{R_2}\right) \end{aligned} \tag{14}$$

   2. The solution for $t > t_0$

$$\begin{aligned} T_I(t,x,r) &= 2\frac{q}{\lambda}\left\{2\left(\frac{kt}{\pi}\right)^{\frac{1}{2}} e^{-\frac{x^2}{4kt}} - x\Phi^*\left(\frac{x}{2\sqrt{kt}}\right) - 2\left(\frac{k(t-t_0)}{\pi}\right)^{\frac{1}{2}} e^{-\frac{x^2}{4k(t-t_0)}} + \right. \\ &\left. + x\Phi^*\left(\frac{x}{2\sqrt{k(t-t_0)}}\right)\right\} - \frac{qR_2}{\lambda}\sum_{i=1}^{\infty}\left\{e^{-\frac{\xi_i}{R_2}x}\left[\Phi\left(\frac{\xi_i}{R_2}\sqrt{kt} - \frac{x}{2\sqrt{kt}}\right) + 2\right] + \right. \\ &+ e^{\frac{\xi_i}{R_2}x}\Phi\left(\frac{\xi_i}{R_2}\sqrt{kt} + \frac{x}{2\sqrt{kt}}\right) - e^{-\frac{\xi_i}{R_2}x}\left[\Phi\left(\frac{\xi_i}{R_2}\sqrt{k(t-t_0)} - \frac{x}{2\sqrt{k(t-t_0)}}\right) + 2\right] - \\ &\left. - e^{-\frac{\xi_i}{R_2}x}\Phi\left(\frac{\xi_i}{R_2}\sqrt{k(t-t_0)} - \frac{x}{2\sqrt{k(t-t_0)}}\right)\right\} x \frac{\frac{R_1}{R_2}}{\left(\frac{\alpha R_2}{\lambda}\right)^2 + \xi^2} \frac{J_1\left(\xi_i \frac{R_1}{R_2}\right)}{J_0(\xi_i)} J_0\left(\xi_i \frac{r}{R_2}\right) \end{aligned} \tag{15}$$

where $\{\xi_i\}$ are the roots of the equation $\frac{\alpha}{\lambda} J_0(\xi) - \frac{\xi}{R_2} J_1(\xi) = 0$, $\Phi^*(x) = 1 - \Phi(x)$ and $\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\xi^2} d\xi$

$R_1/R_2 = 1$

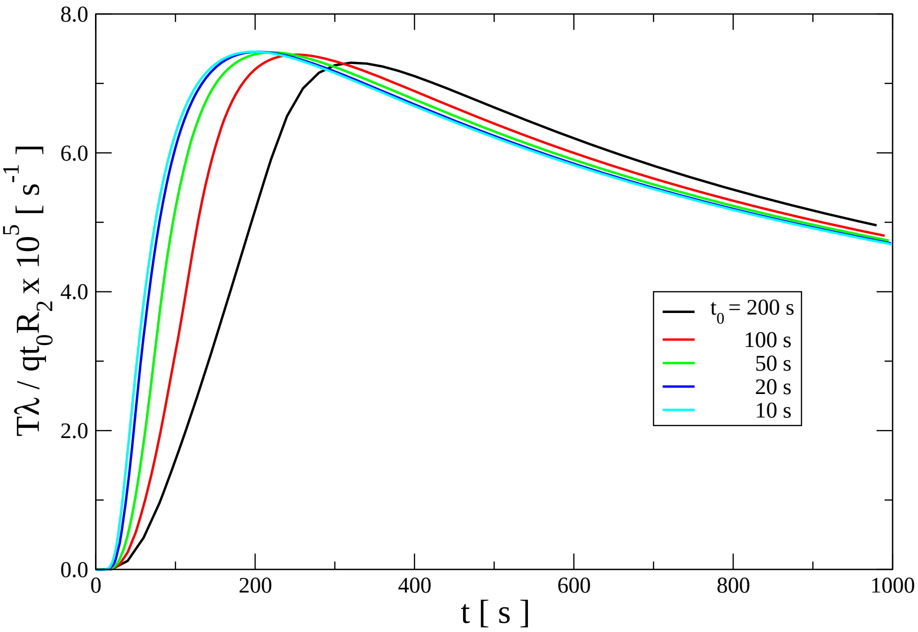

Fig 2 Time dependence of temperature for different pulse duration t_0

$R_1/R_2 = 1/2$

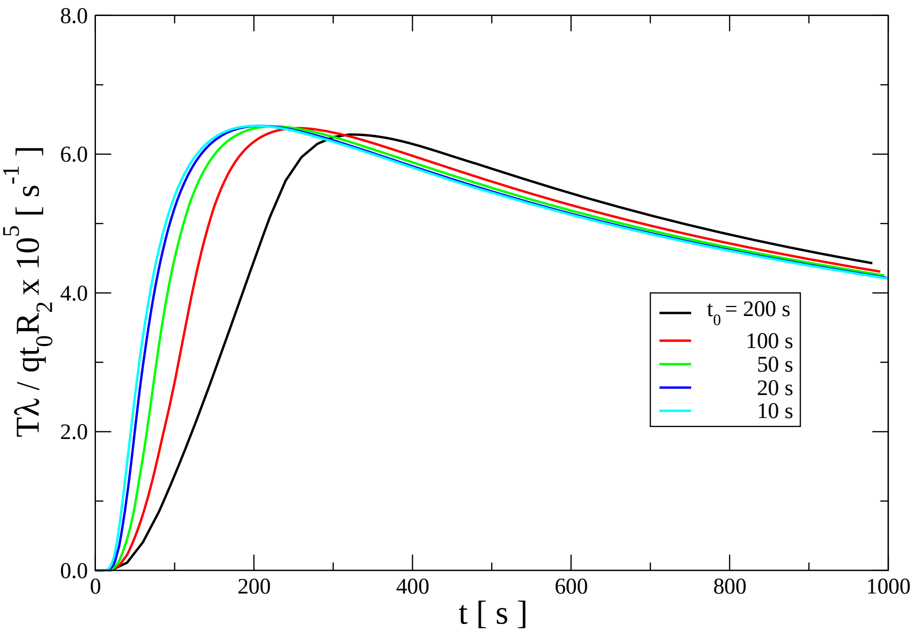

Fig 3 Time dependence of temperature for different pulse duration t_0

$R_1/R_2 = 1$

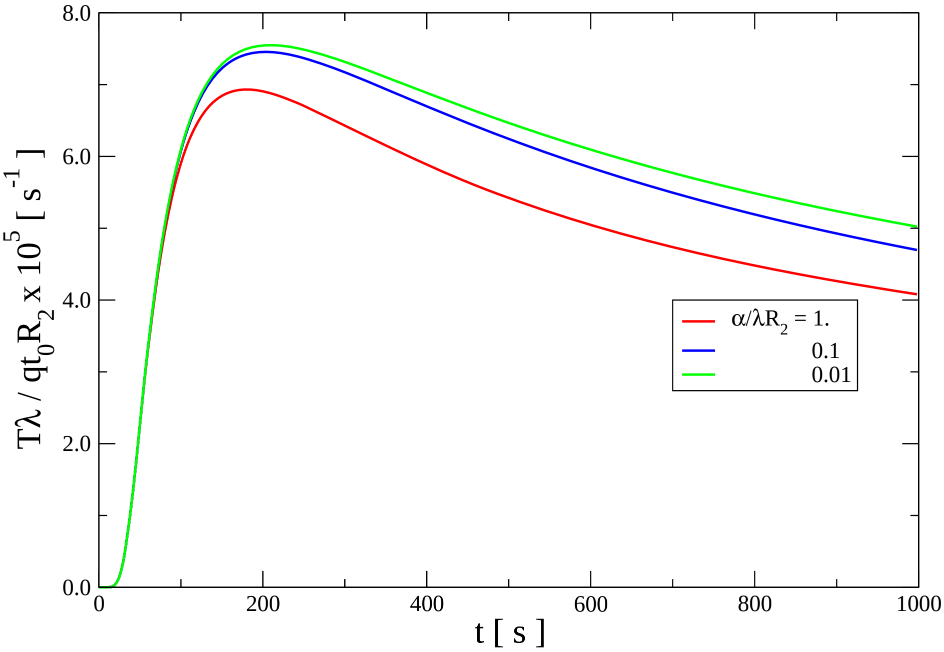

Fig 4 Time dependence of temperature for different coefficient of surface heat loss

$R_1/R_2 = 1/2$

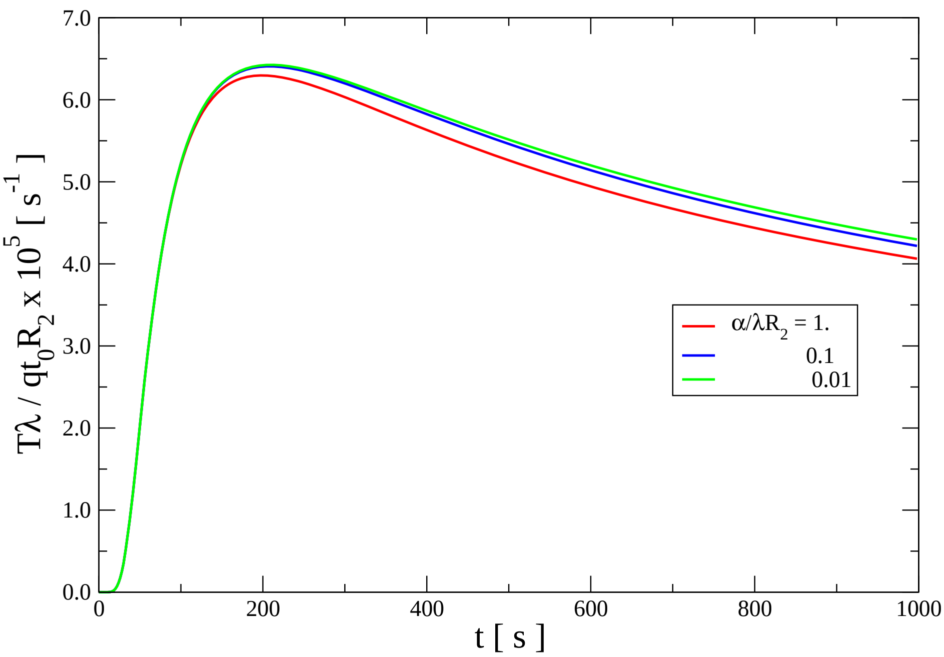

Fig 5 Time dependence of temperature for different coefficient of surface heat loss

3 Analysis

For obtaining the information about the thermo-physical parameters of the sample it is necessary to know the sample temperature field. The known temperature field makes it possible to acquire thermo-physical parameters by the following way: Firstly one measures the temperature in the definite place (on the axis of the sample) according to time. Secondly the experimental data should be optimally fitting by the theoretical time dependence of the temperature. From the best fitted one gets the values of the sample thermo-physical parameters.

The theoretical relation of the time dependence of the temperature was analyzed in two cases: In the first case the duration of the heat pulse has been changing. It was found that there exists the heat pulse duration below which the form of time dependence does not change. In the second case it was analyzed the influence of the heat transfer from sample surface on the temperature field for different radii of the heat source. It was established that with decreasing radius of the sample the time dependence of the temperature on the axis of the sample depends weekly on the heat transfer from the surface of the sample.

4 Conclusion

At the chosen parameters $\lambda = 0.2\mathrm{JK^{-1}s^{-1}m^{-1}}$, $k = 10^{-6}\mathrm{m^2s^{-1}}$, $\alpha = 10^{-2}\mathrm{Jm^{-2}s^{-1}K^{-1}}$ the temperature field does not depend on t_0 for the heat pulse duration $t_0 < 10$s Fig. 2,3.

For ratio of radii $\frac{R_1}{R_2} < \frac{1}{4}$ temperature field with the sufficient accuracy does not depend on the boundary condition at $r = R_2$ Fig. 4,5. That is the reason why radius R_1 was chosen smaller than R_2.

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